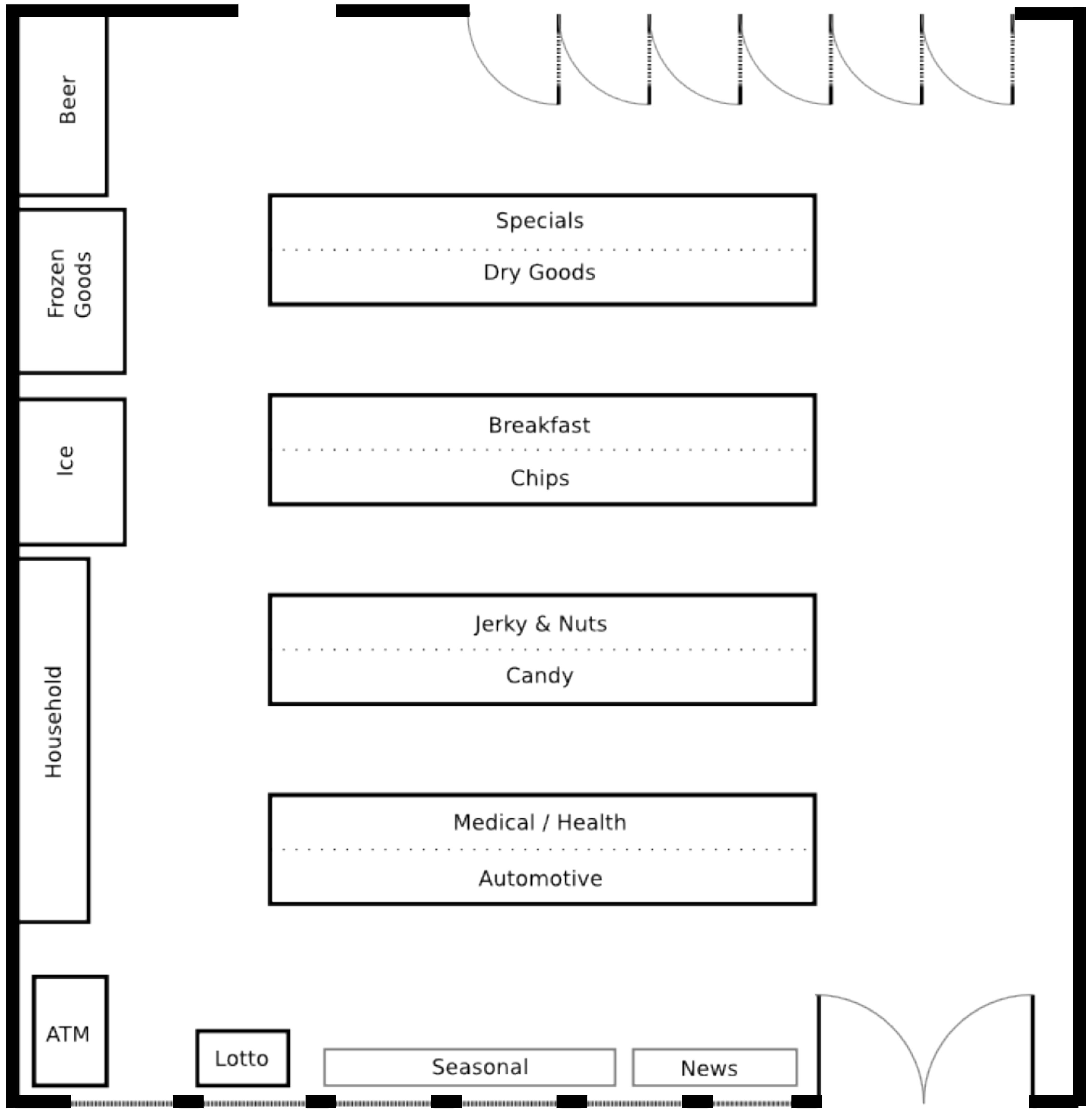
Technische Universität Braunschweig

New Variants of the Floodlight Problem

Bachelor's Thesis

Yannic Lieder, September 5, 2018

Introduction



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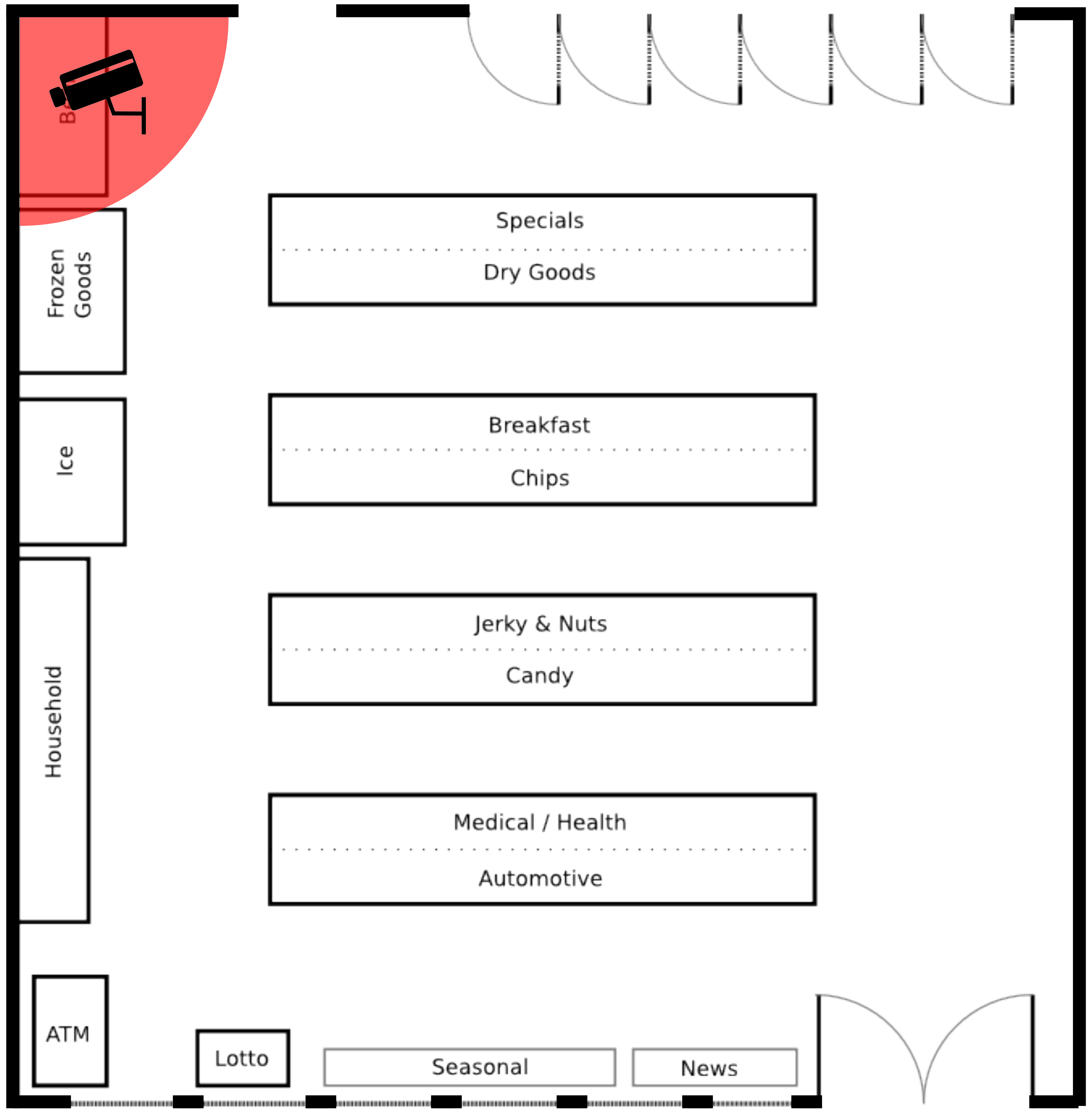

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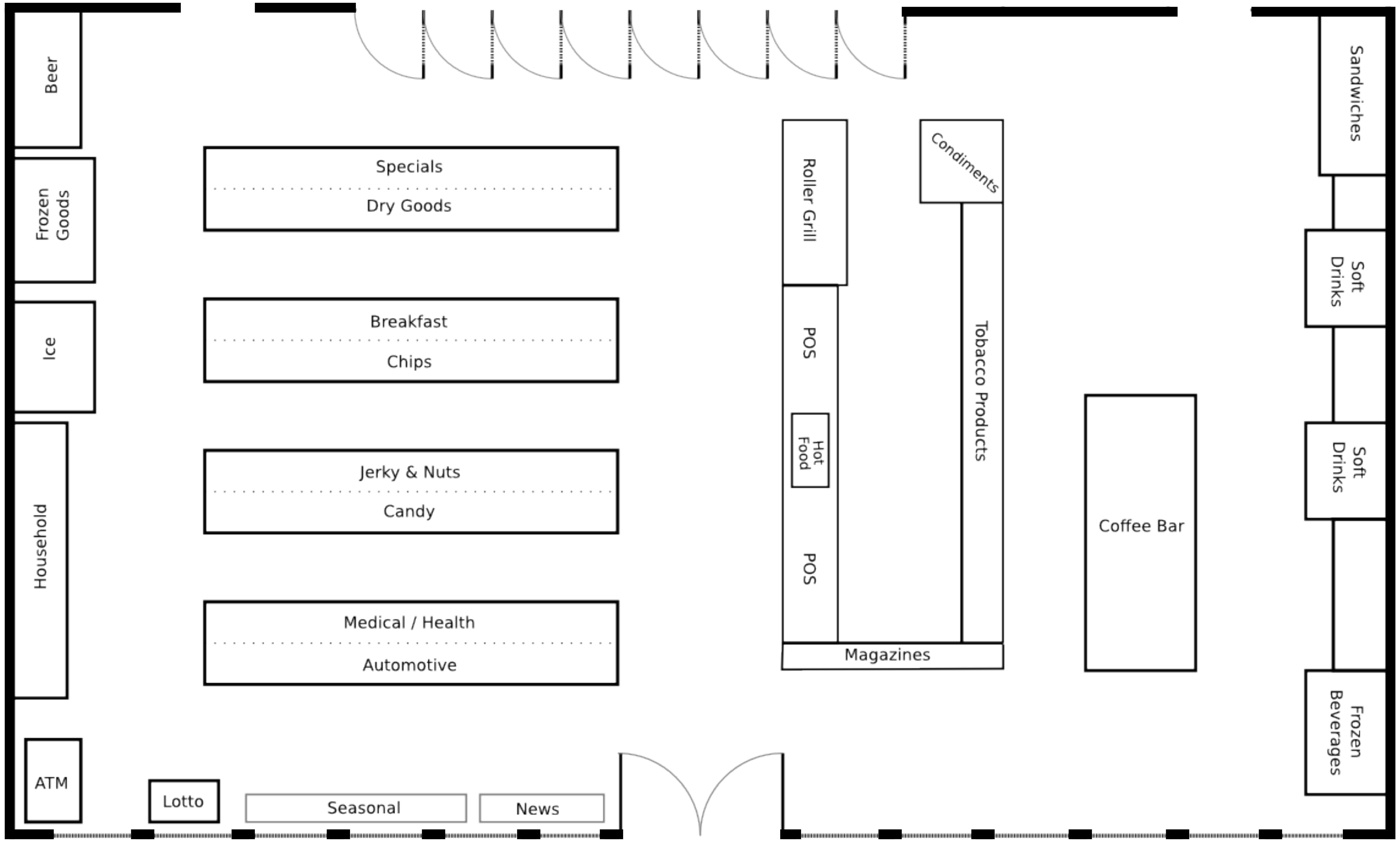

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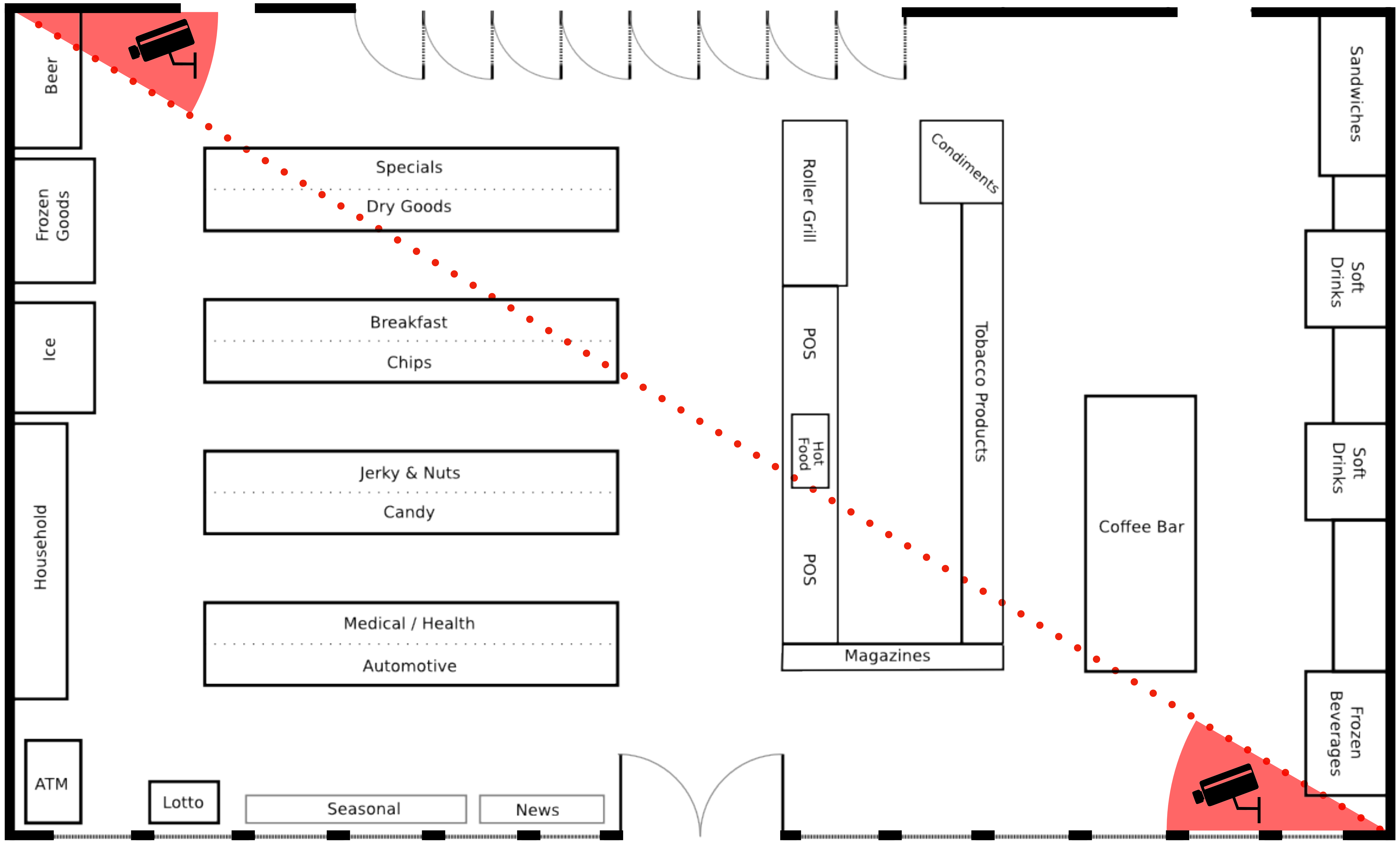
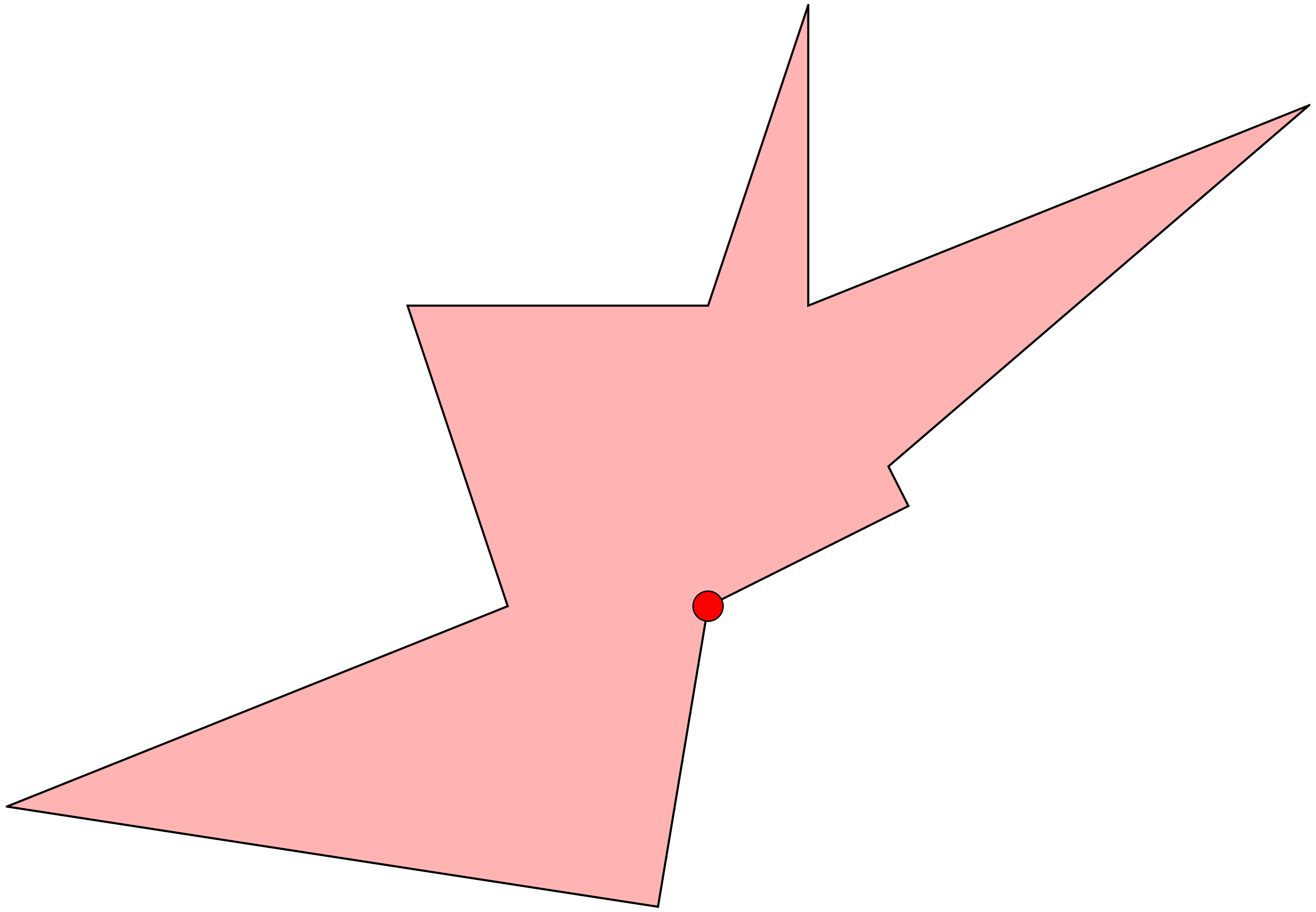


Background

1973 The Art Gallery Problem



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1973 The Art Gallery Problem

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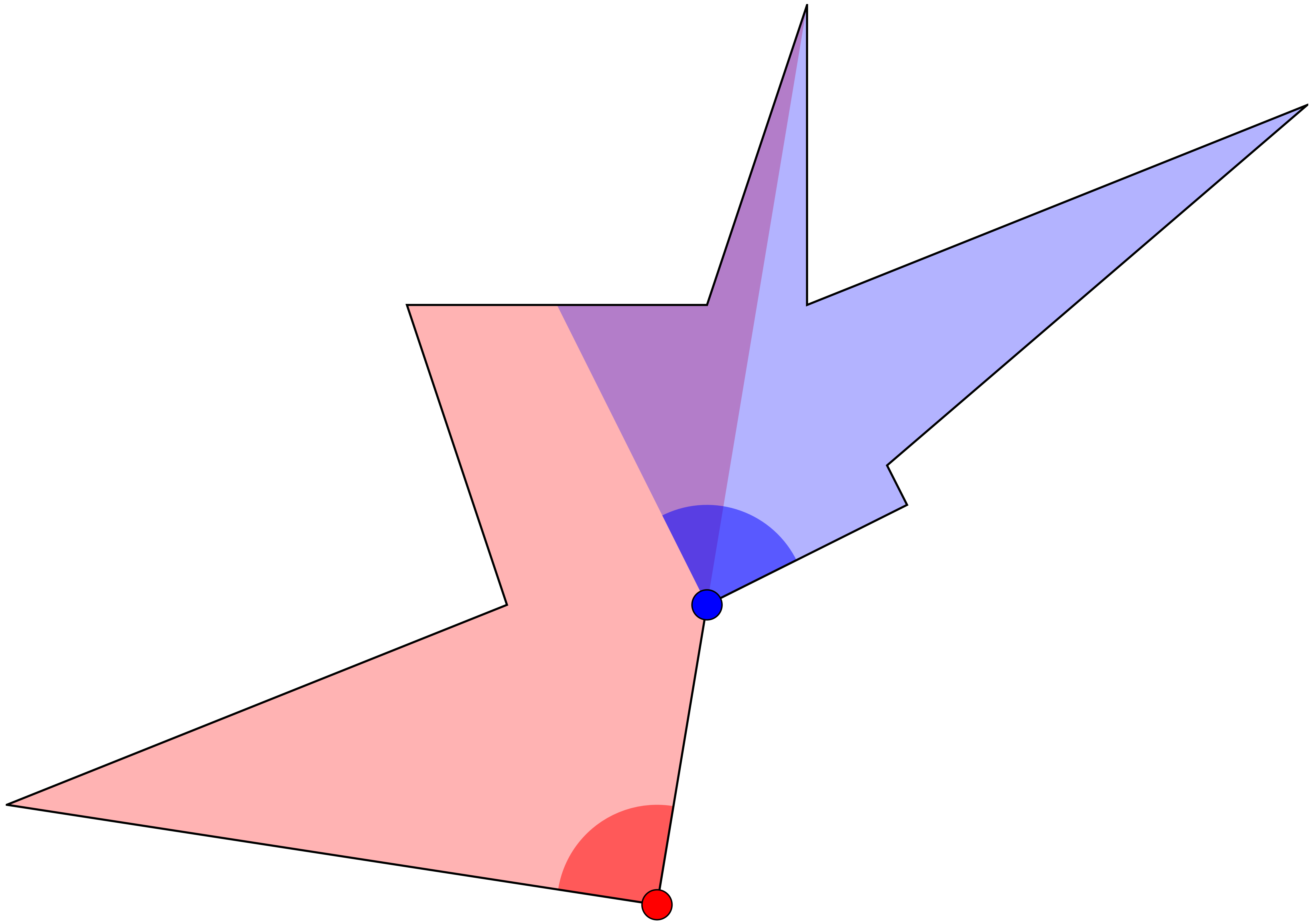
1992 The α- Floodlight Problem



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1973 The Art Gallery Problem

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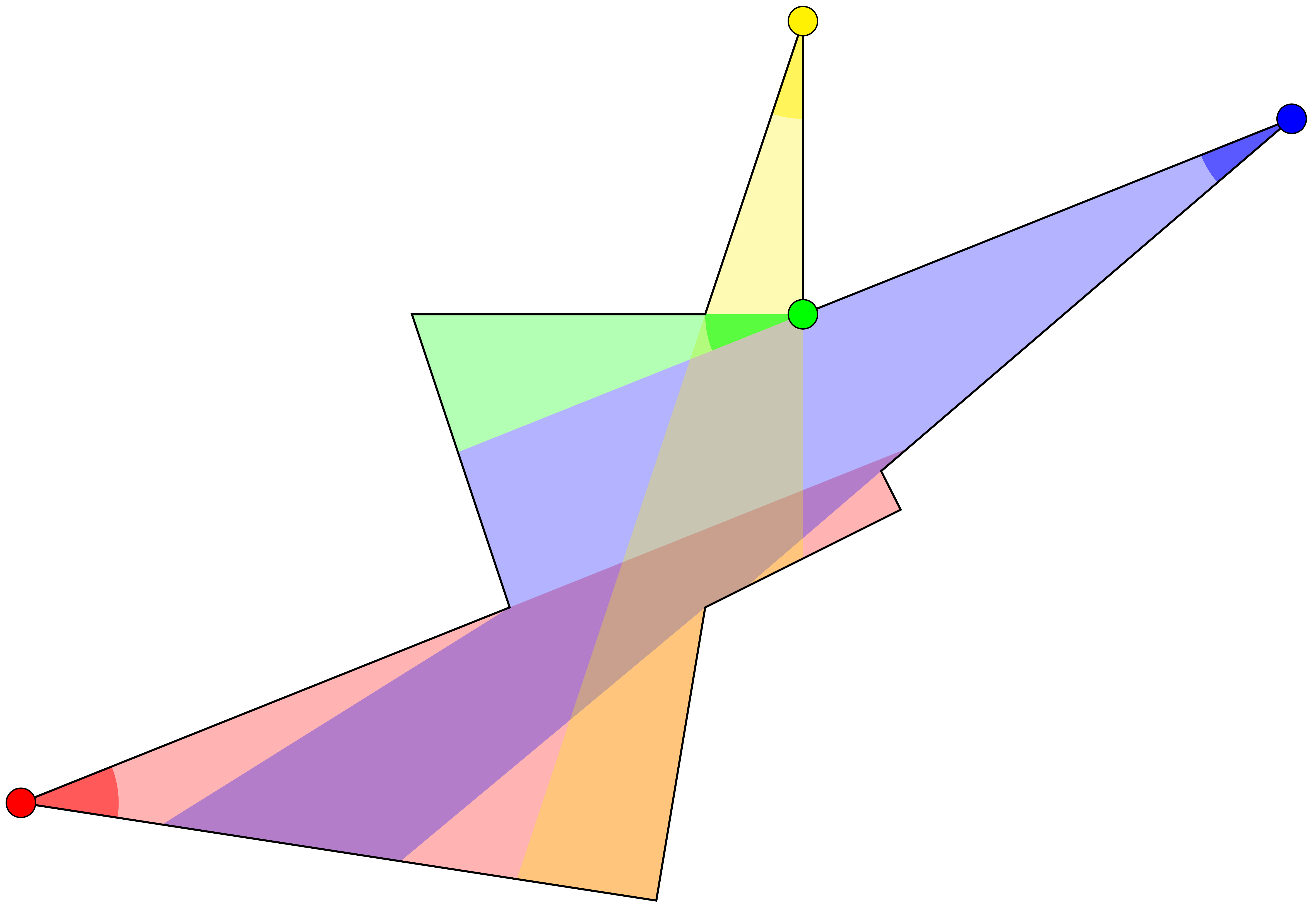
This thesis The Angular Art Gallery Problem



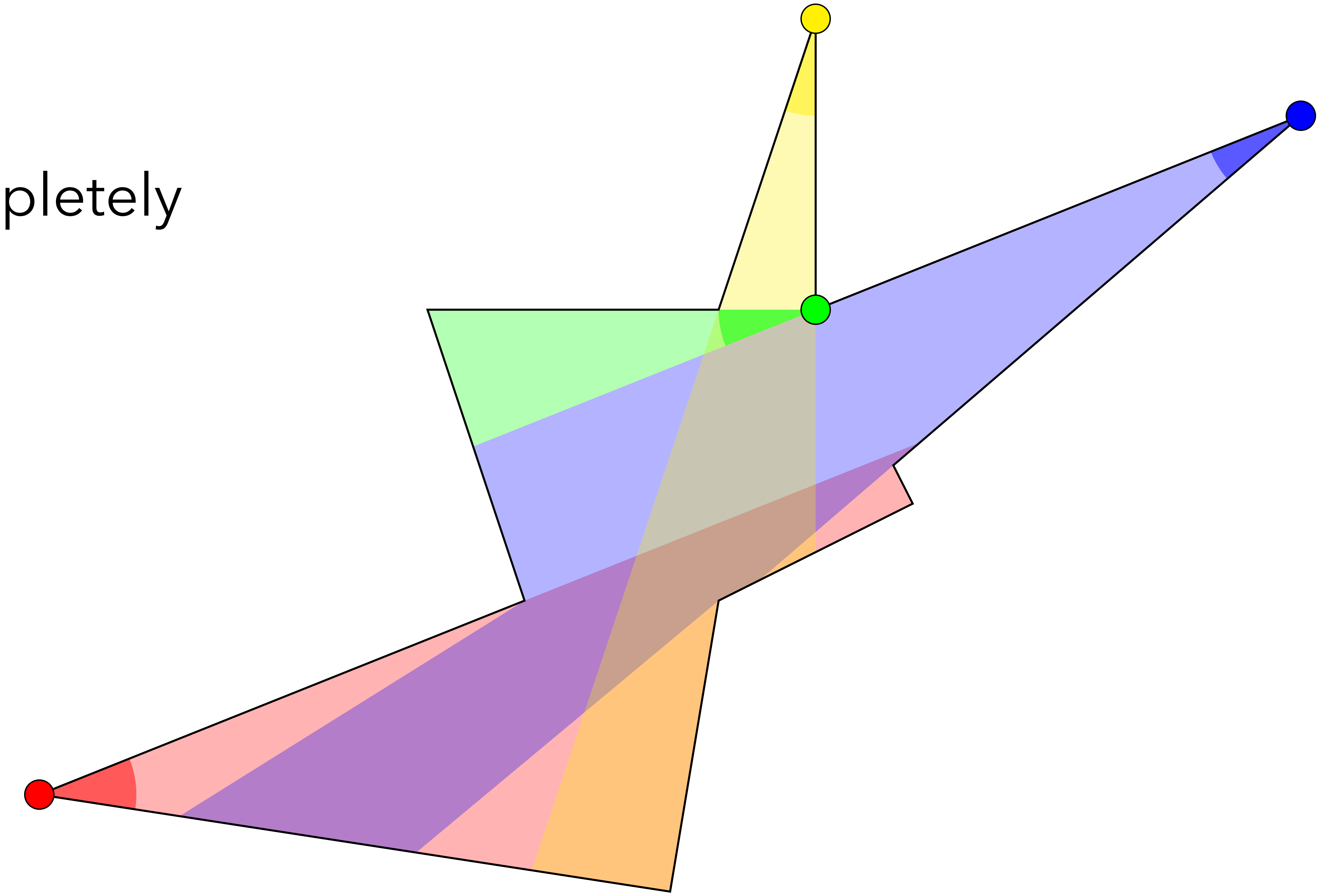
Problem Definition

Angular Art Gallery Problem

Instance: A simple polygon *P*

Wanted: A set of floodlights, covering *P* completely

Minimize: The total angle of all floodlights



Results

	Lower Bound	Upper Bound

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	Lower Bound	Upper Bound
Equilateral Triangles	$\frac{\pi}{3}$	$\frac{\pi}{3}$

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Histograms	$(n-1)\frac{\pi}{6}-\epsilon$	$(n-1)\frac{\pi}{6}$

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Simple Polygons	$(n-1)\frac{\pi}{6} - \epsilon$	$(n-2)\frac{\pi}{4}$

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Duality to independent circle packing

Outline

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Equilateral Triangles

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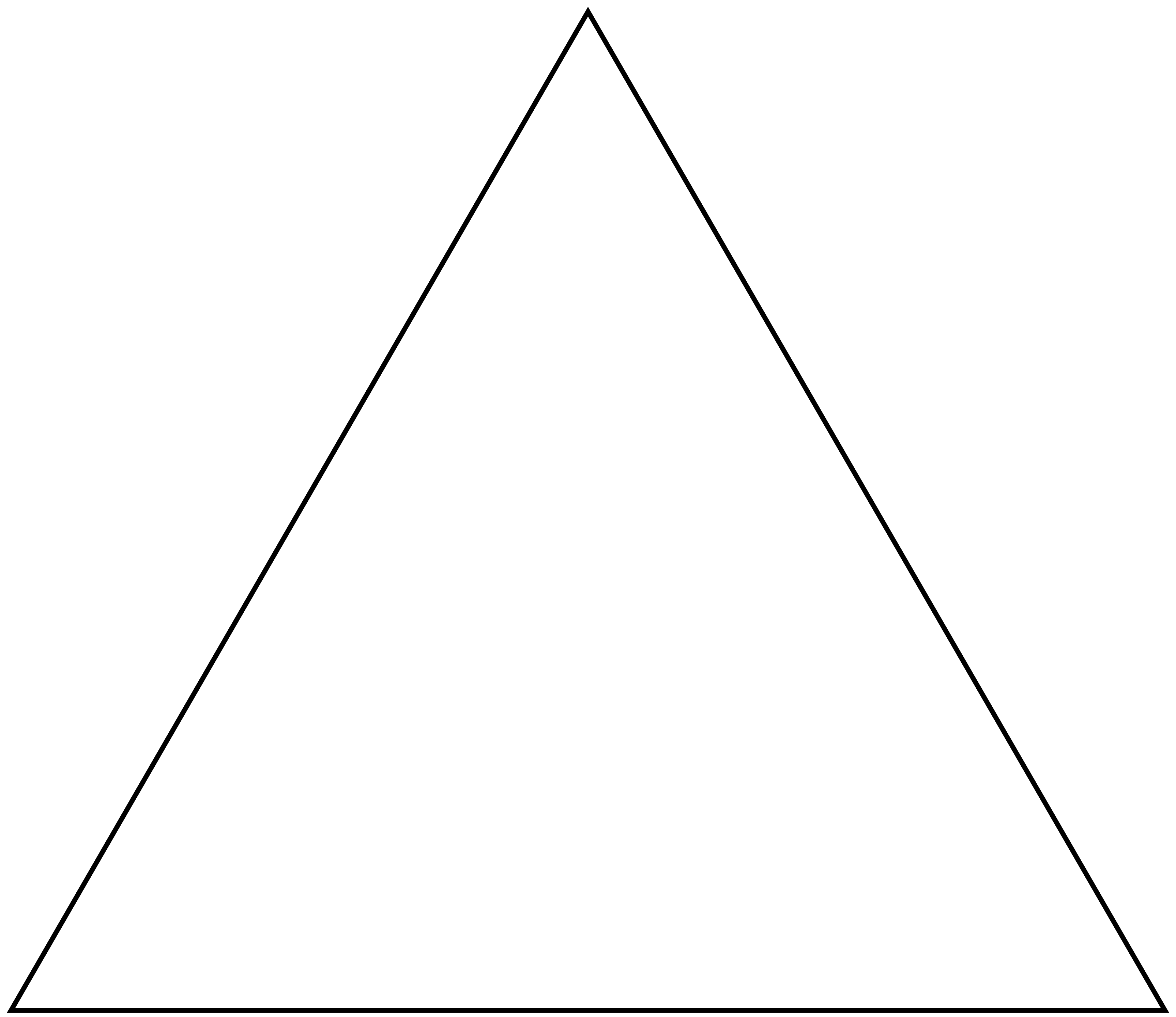

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Optimal Covering of **Equilateral Triangles**

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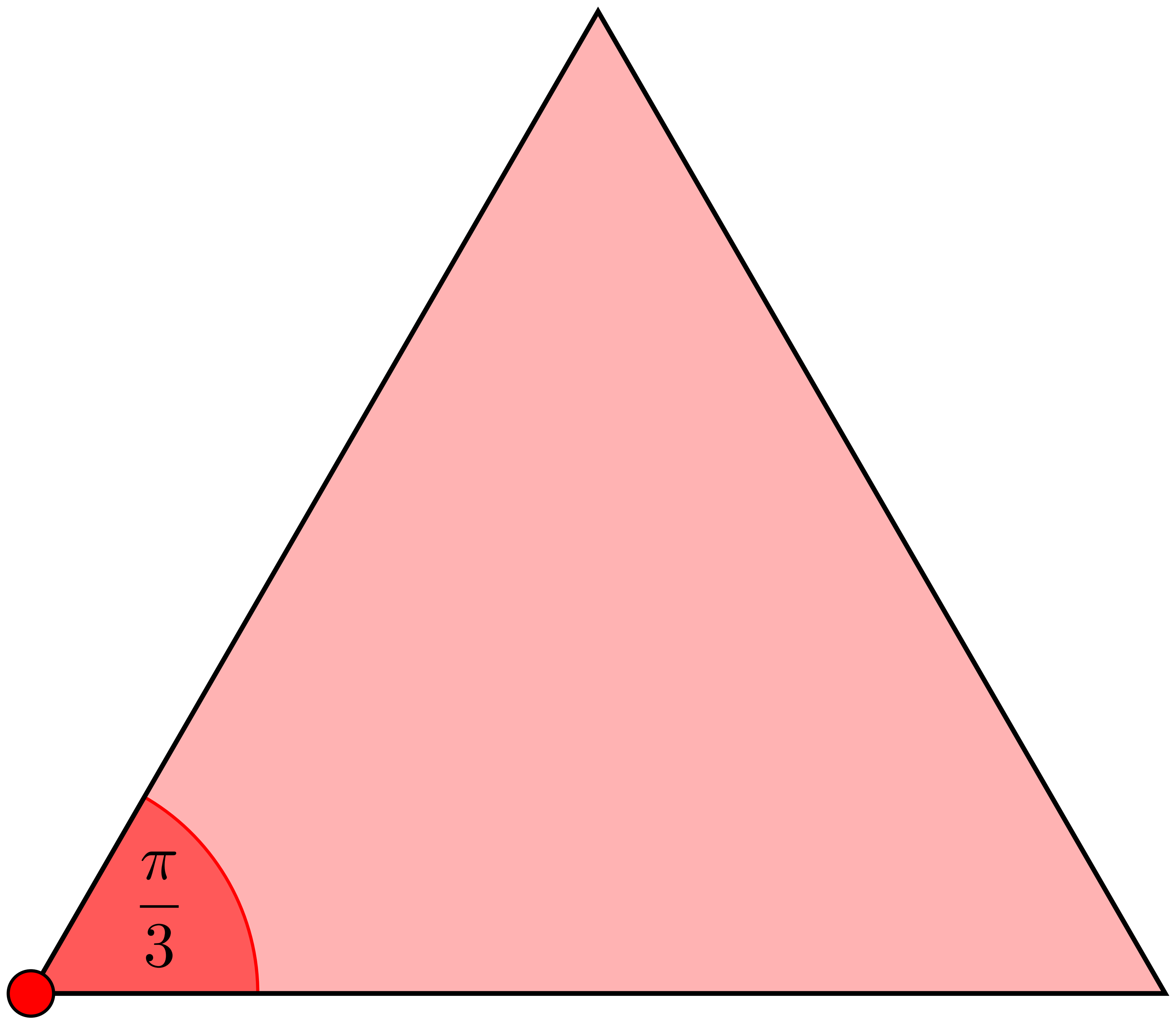
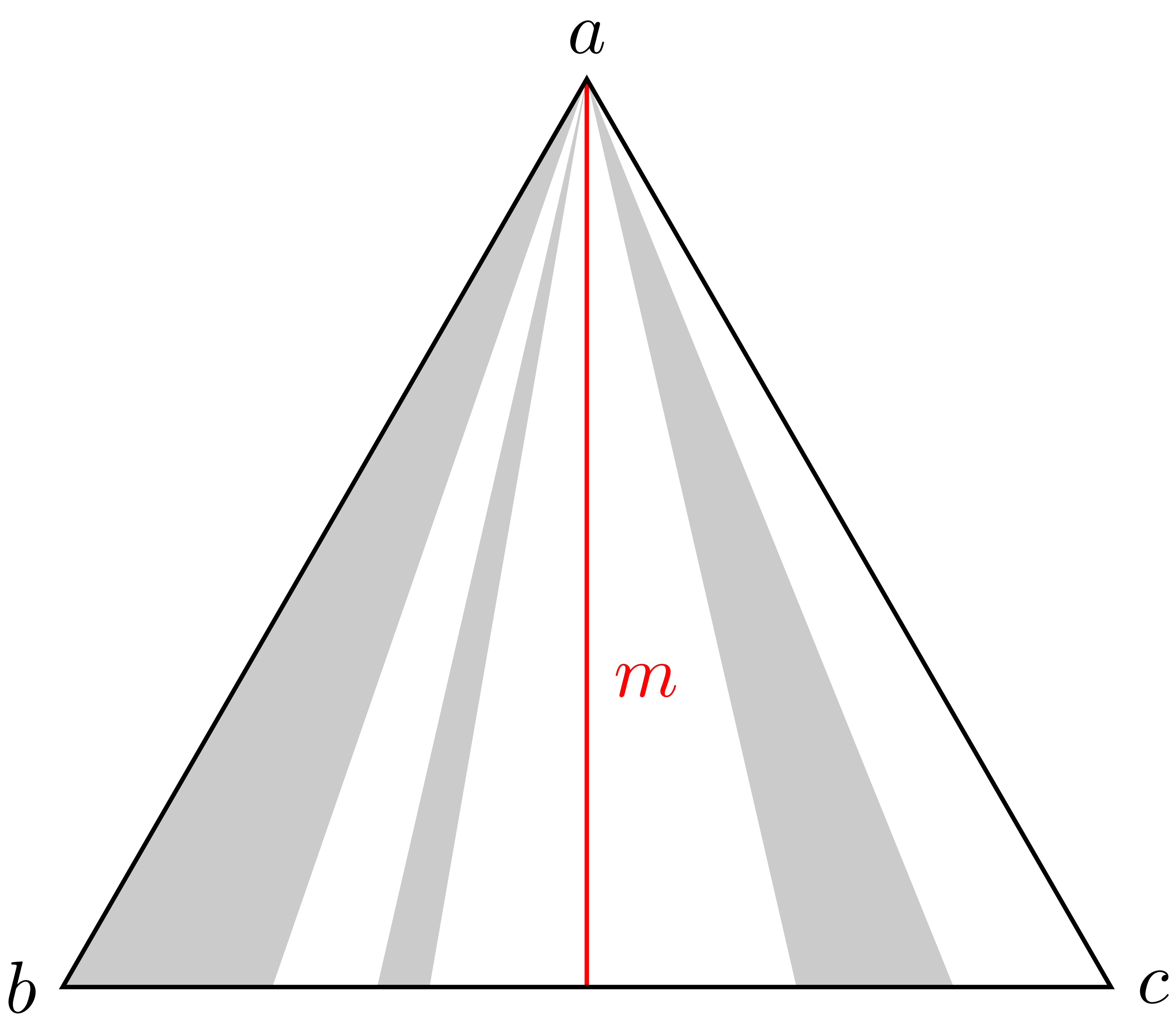
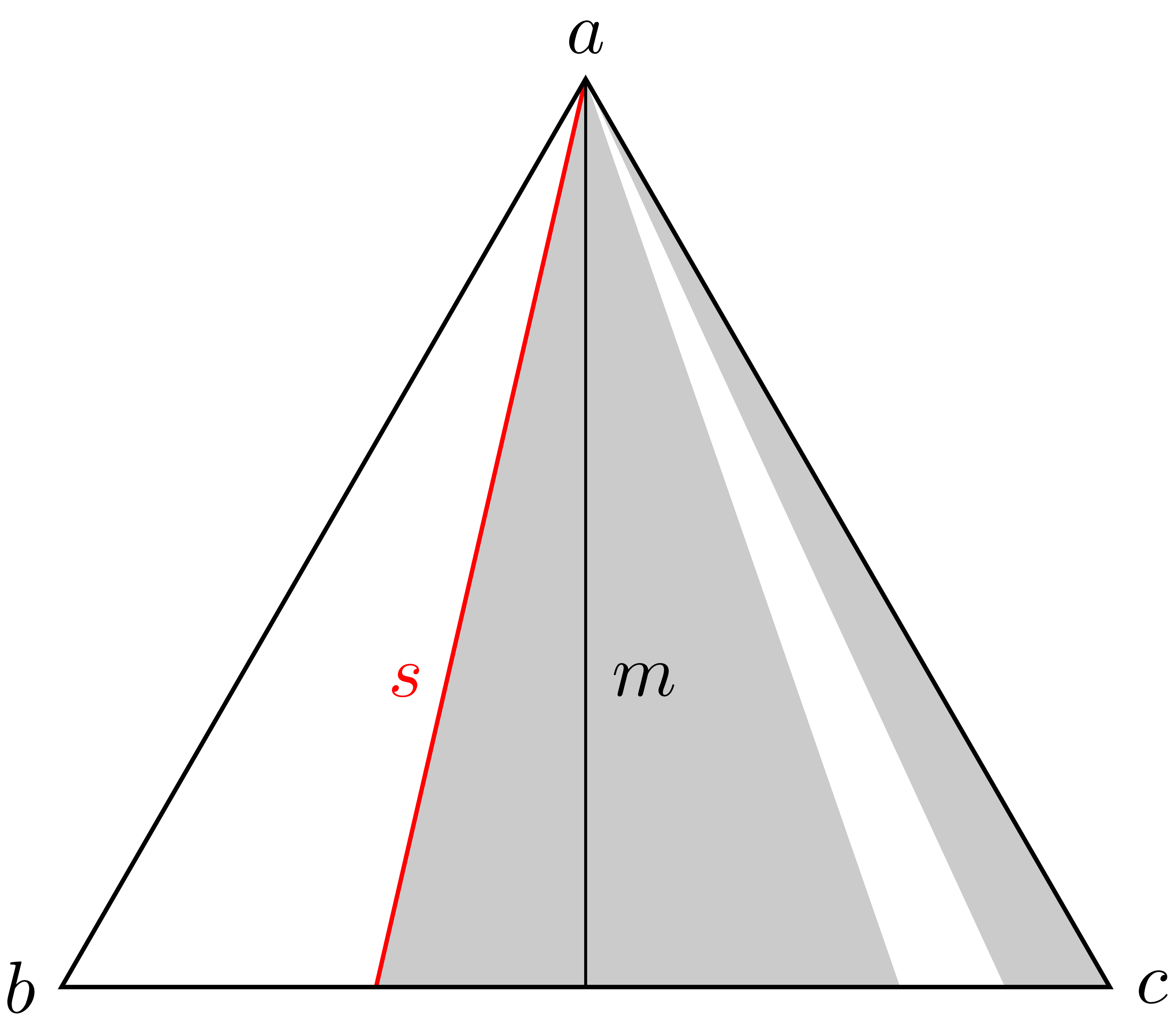


Lower Bound for Equilateral Triangles

Case 1: m is not covered by a



Case 2: m is covered by a



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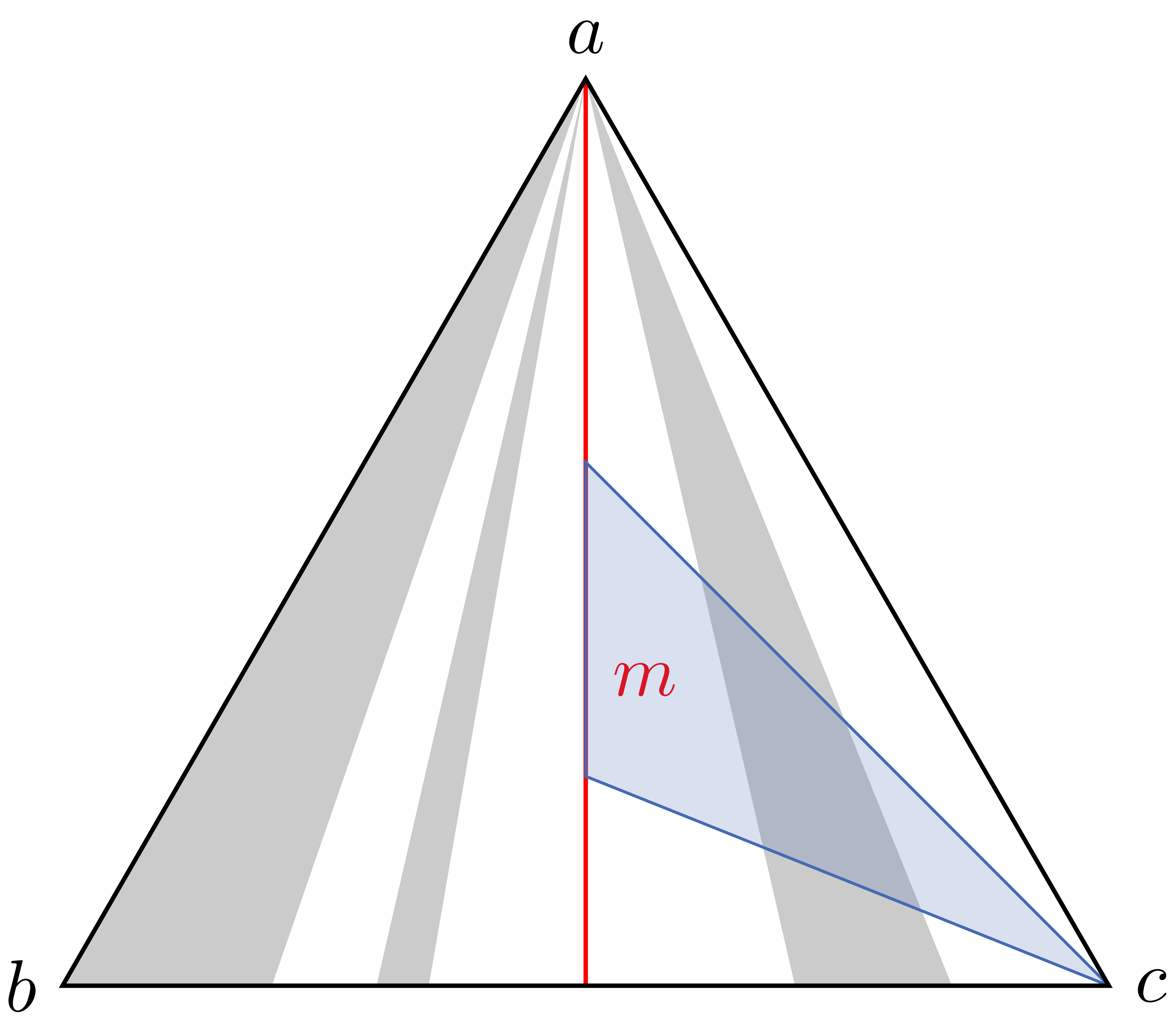

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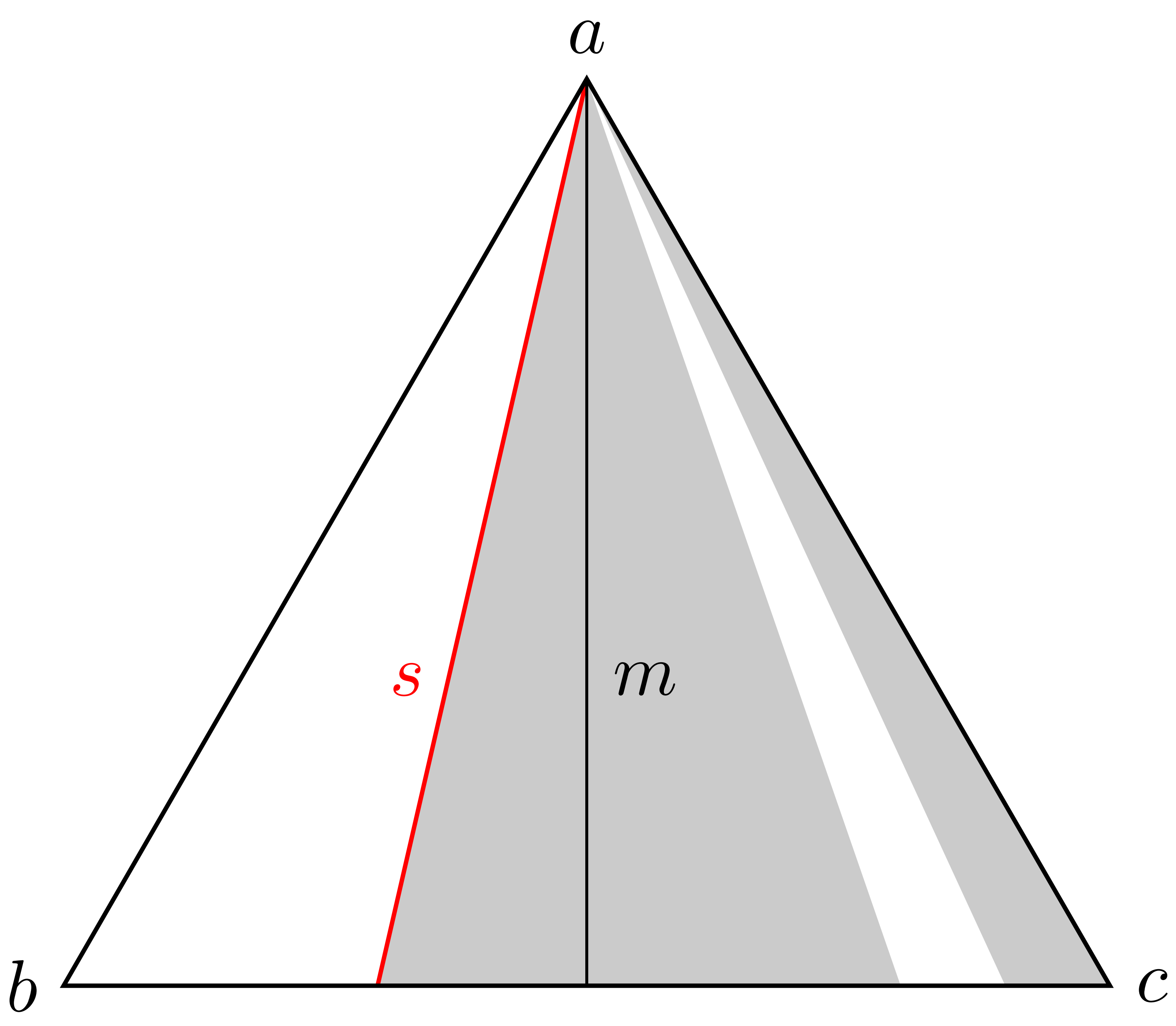

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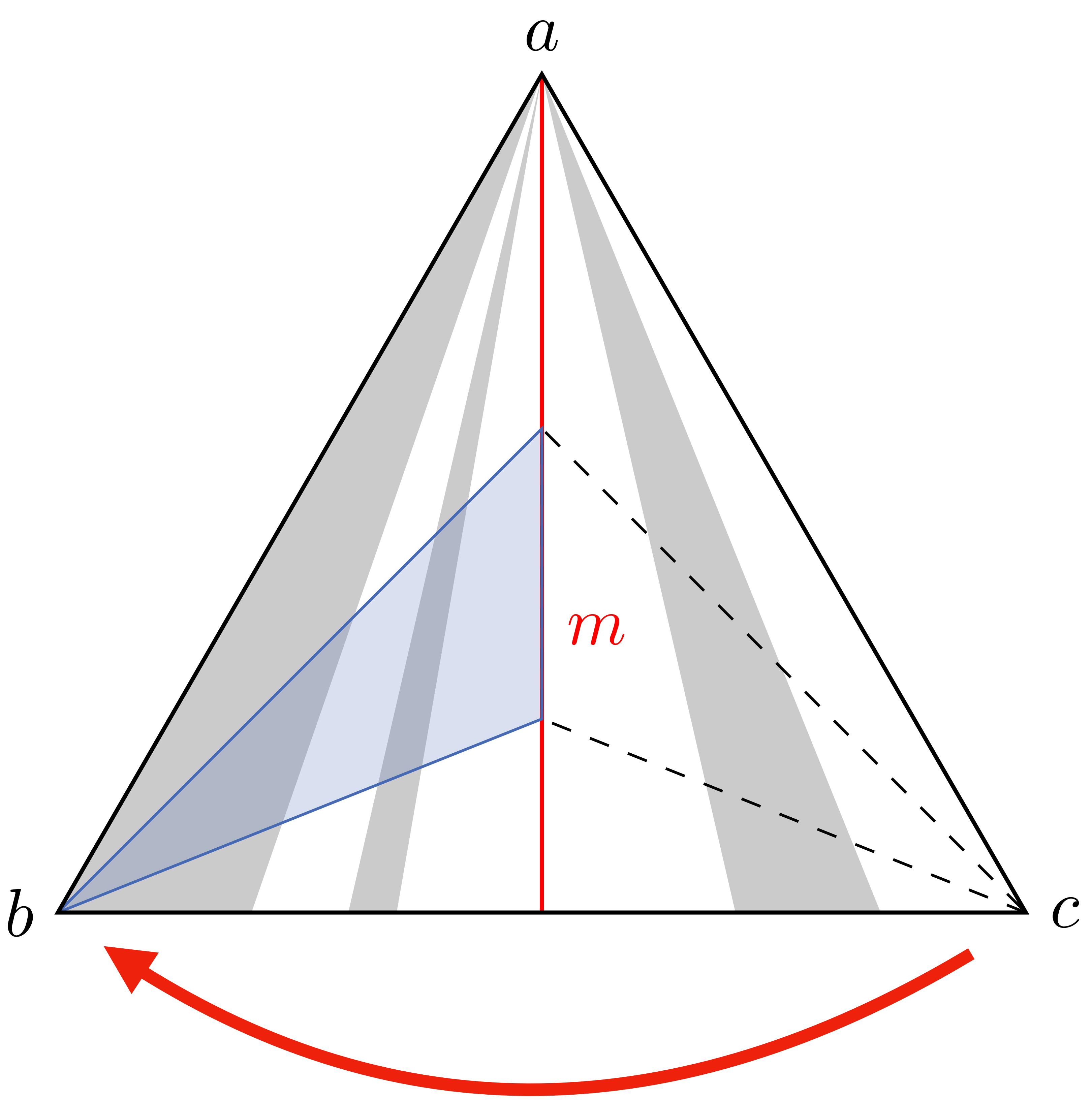

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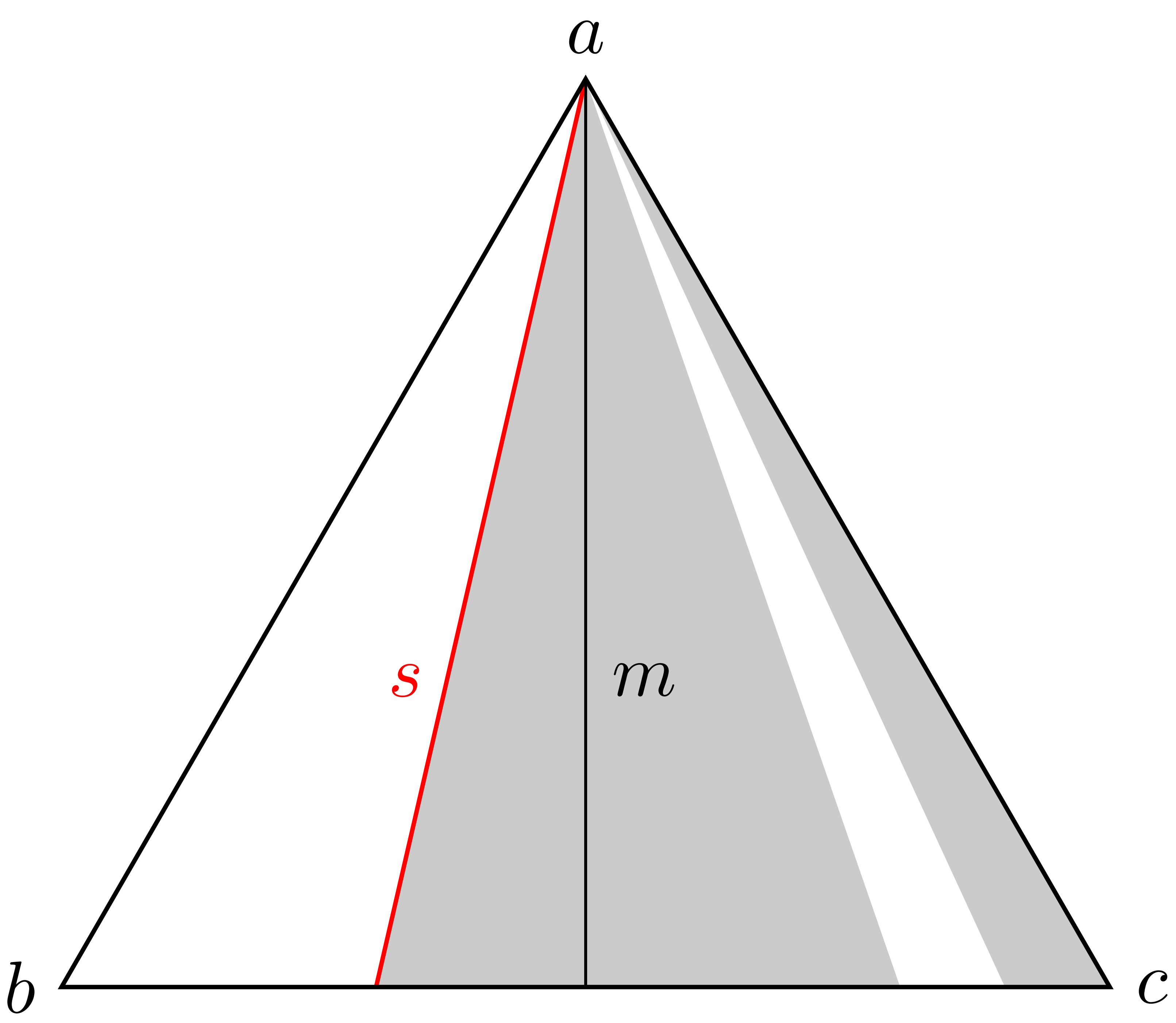

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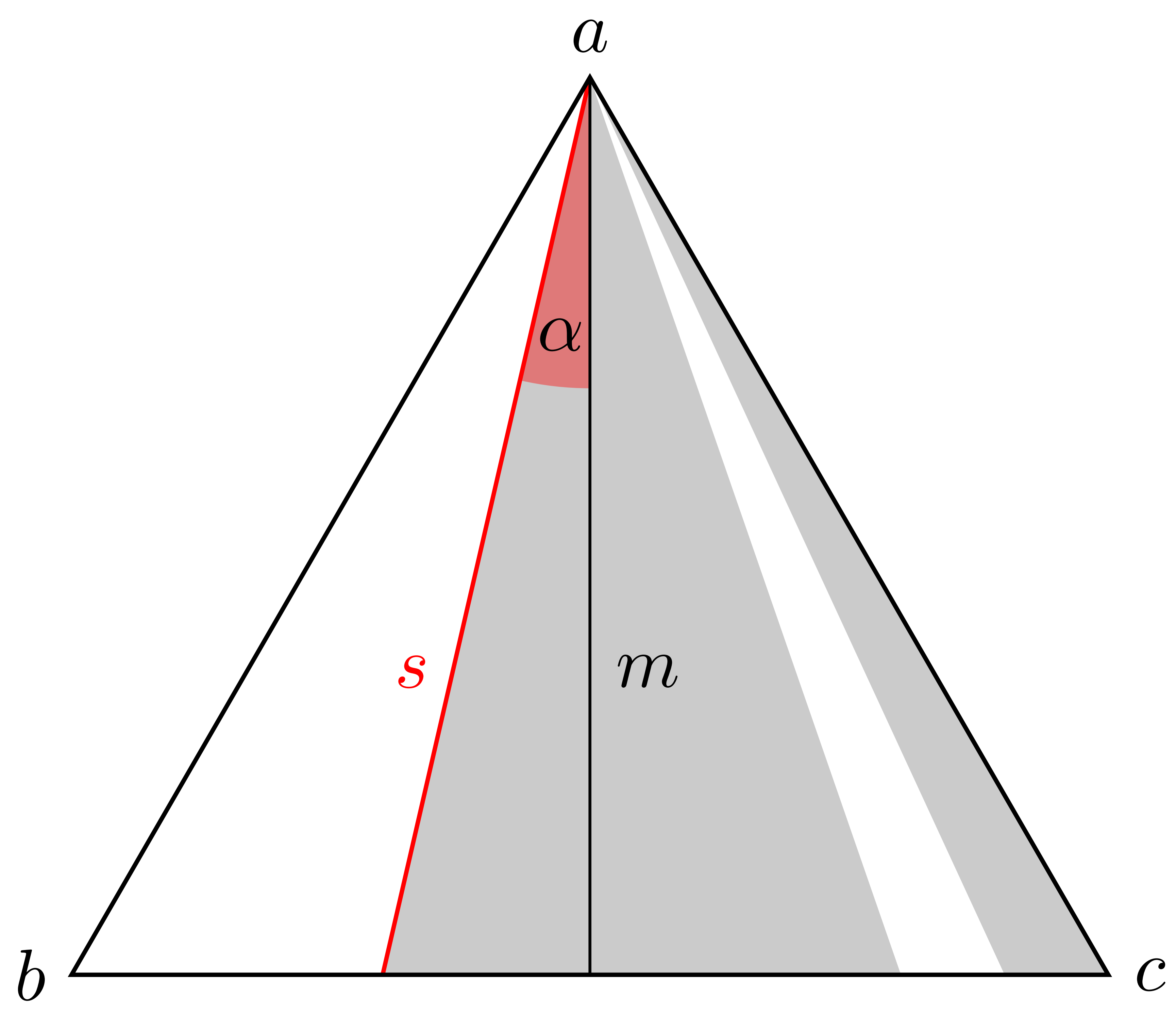

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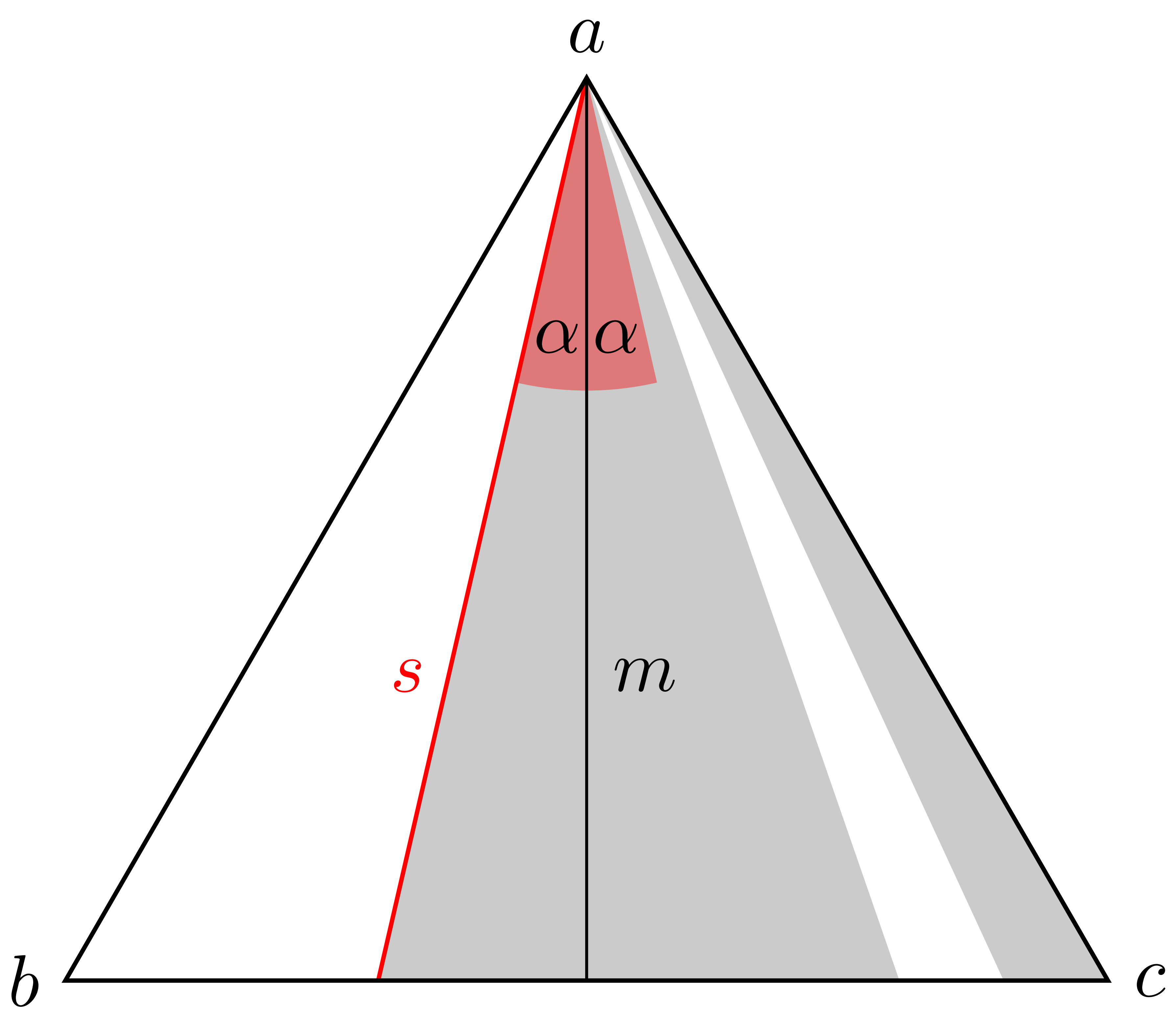

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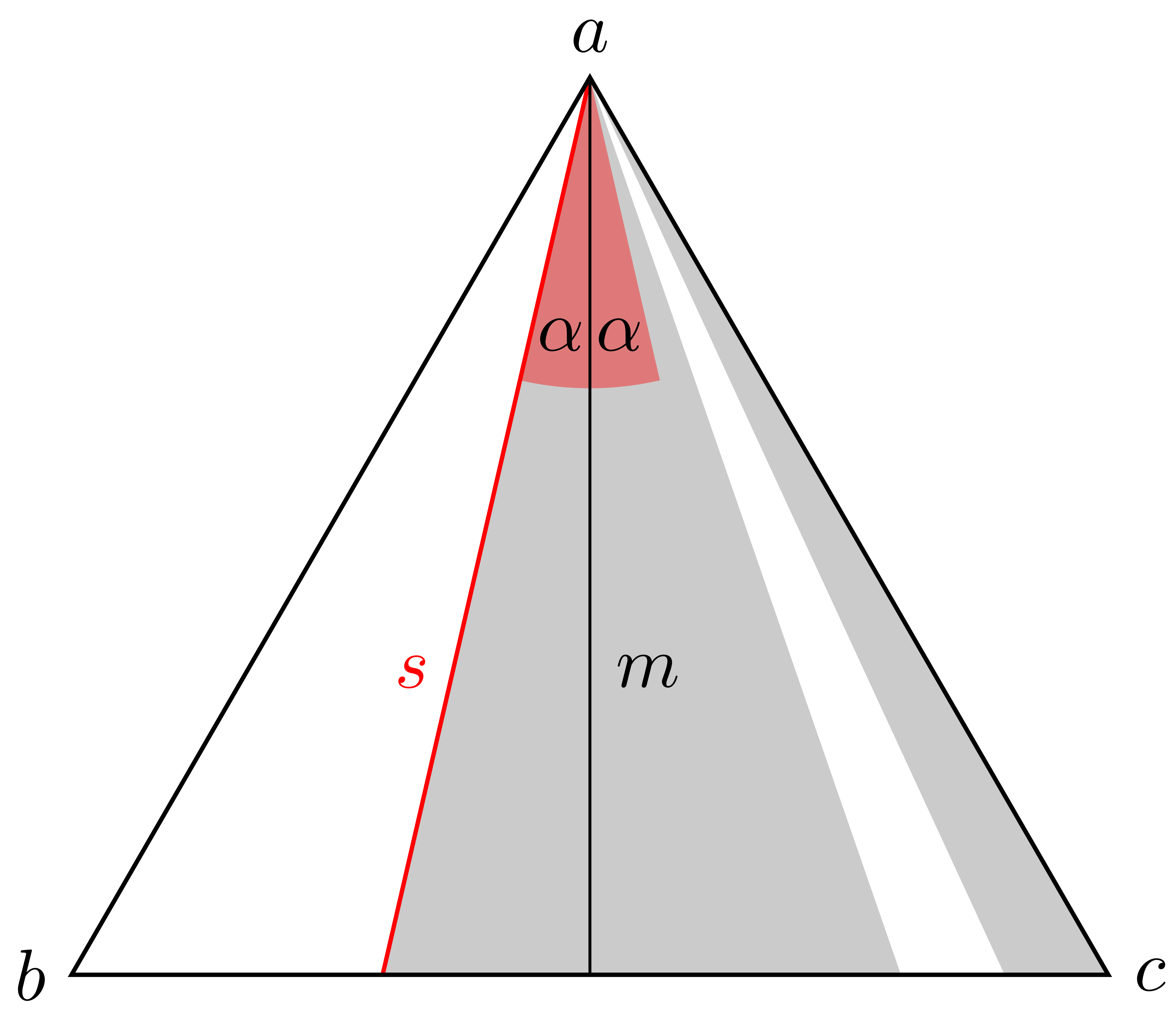
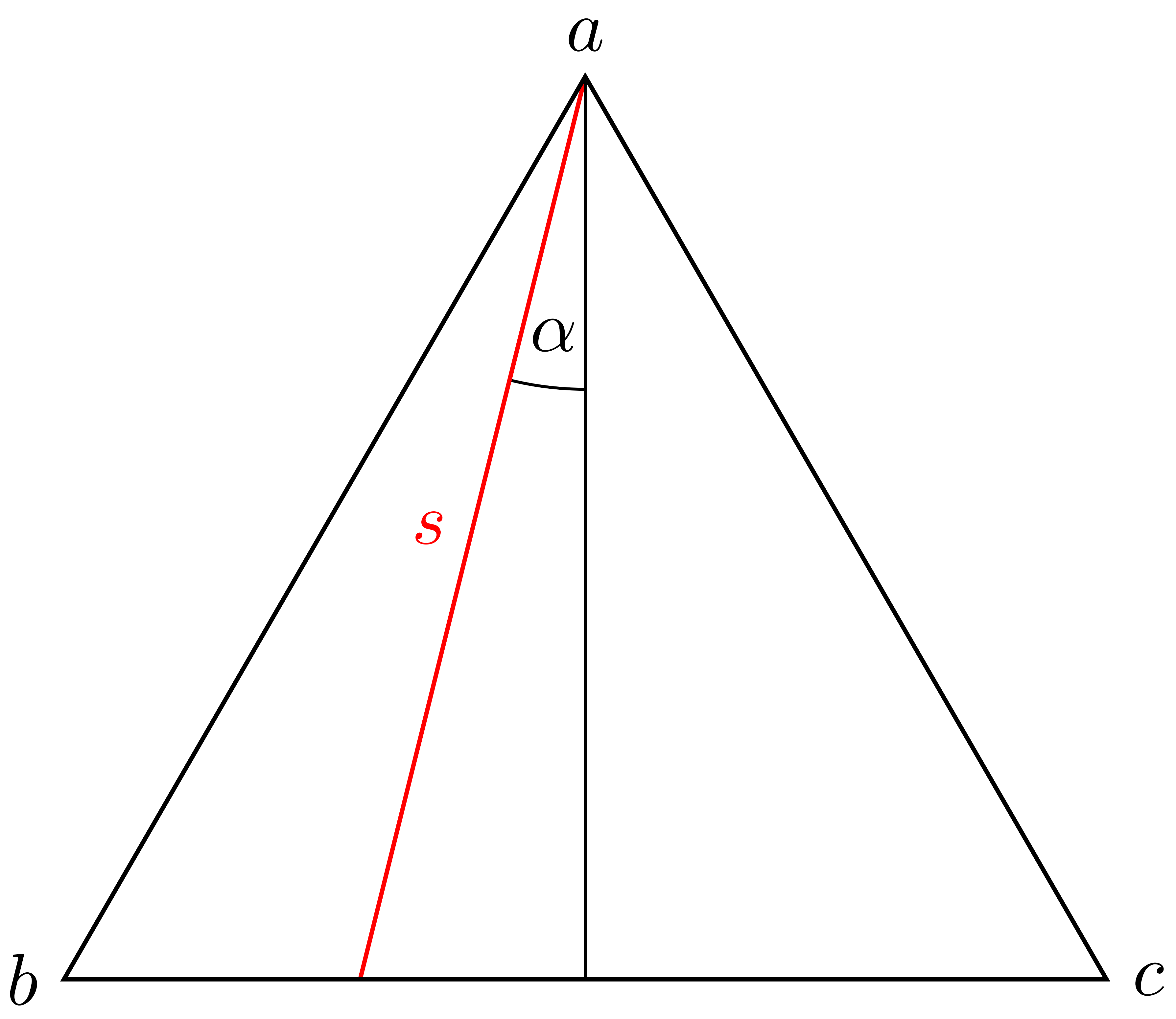


Idea: Showing that an angle of at least

$$\frac{\pi}{3} - 2\alpha$$

is required to cover s.

Minimum Covering of s



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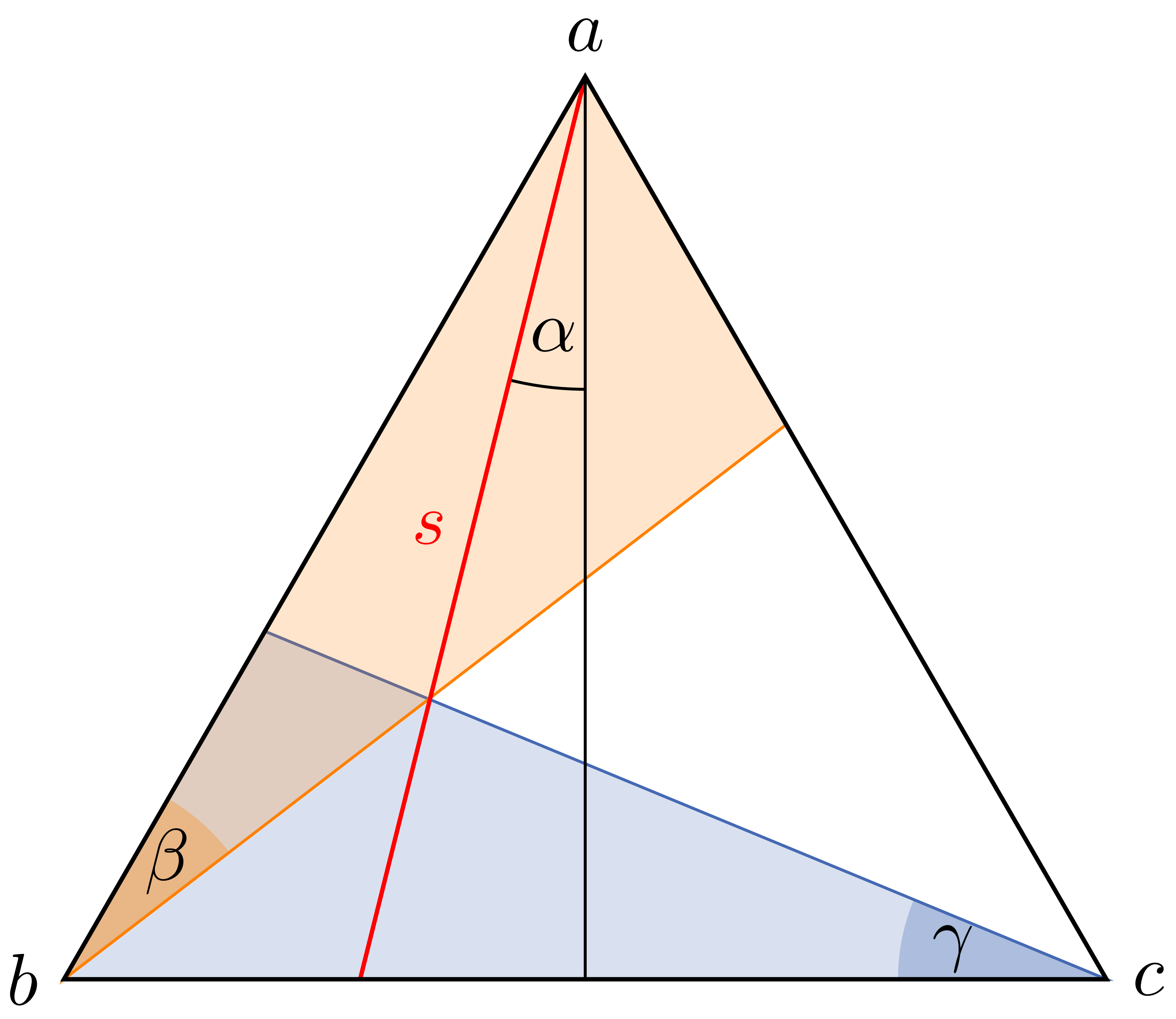

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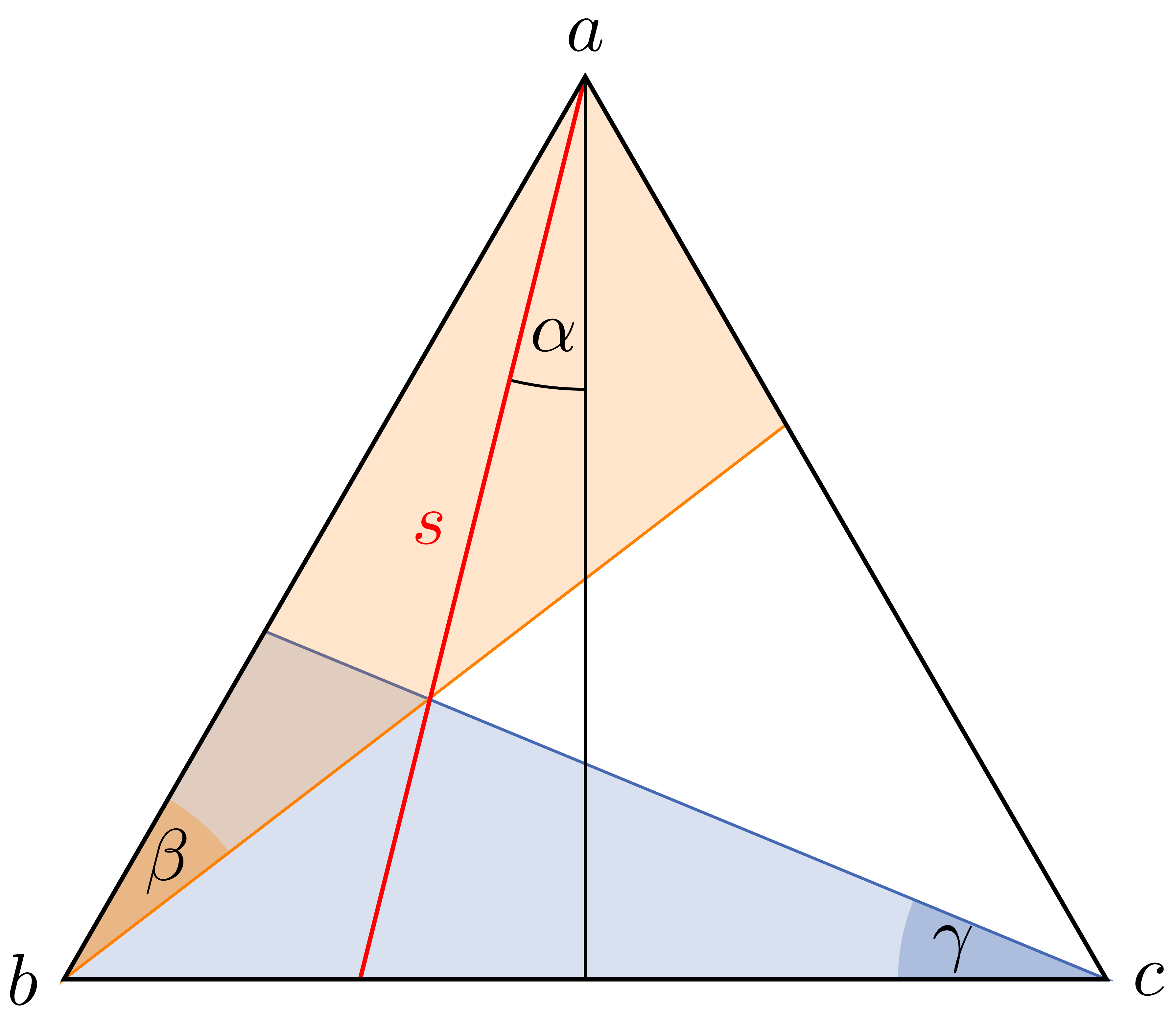

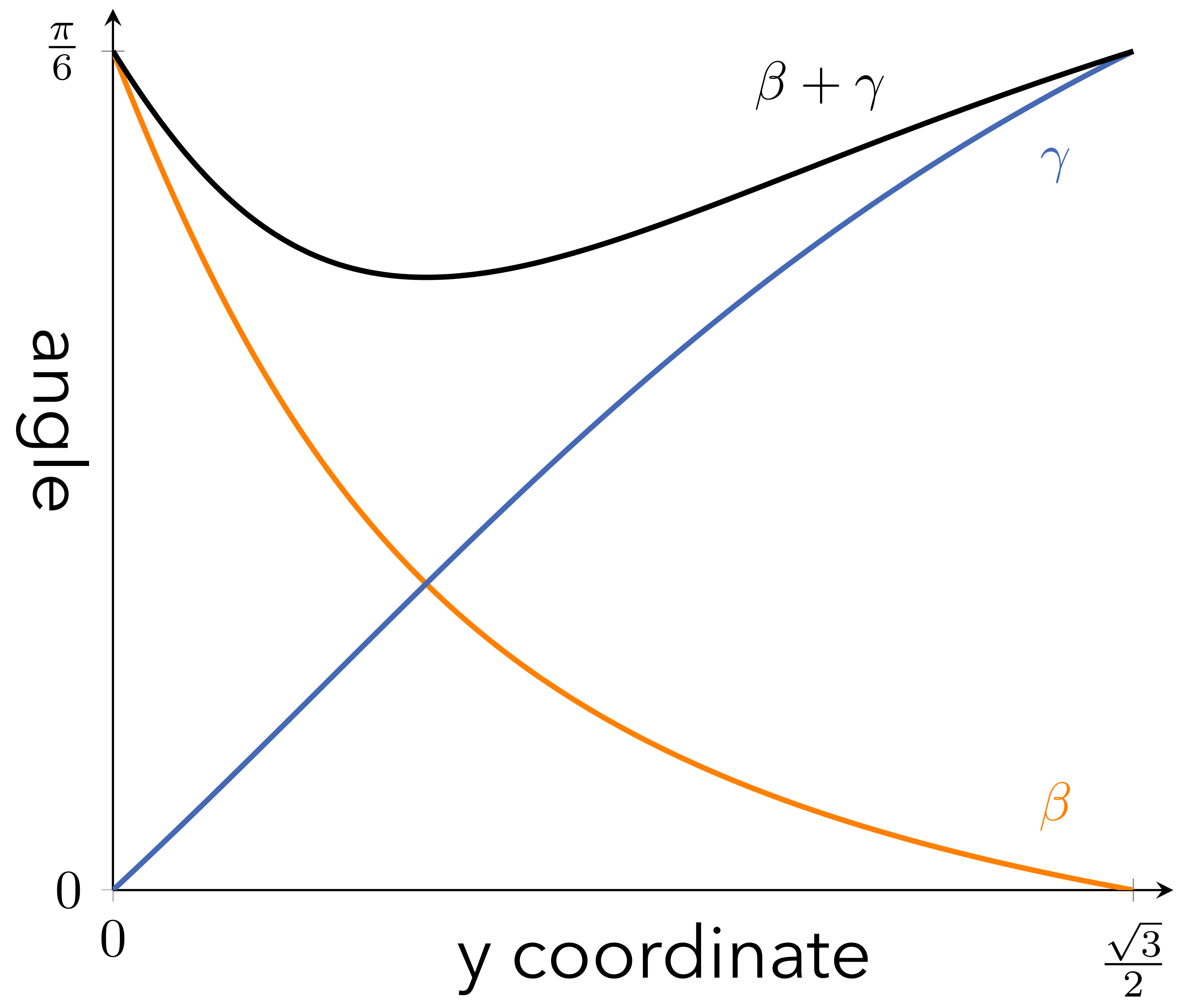

Minimum Covering of s

$$(\beta+\gamma)(y) = \arctan\left(\frac{\frac{1}{2} - \tan(\alpha)\left(\frac{\sqrt{3}}{2} - y\right)}{y}\right) - \frac{\pi}{6} + \arctan\left(\frac{y}{\frac{1}{2} + \tan(\alpha)\left(\frac{\sqrt{3}}{2} - y\right)}\right)$$

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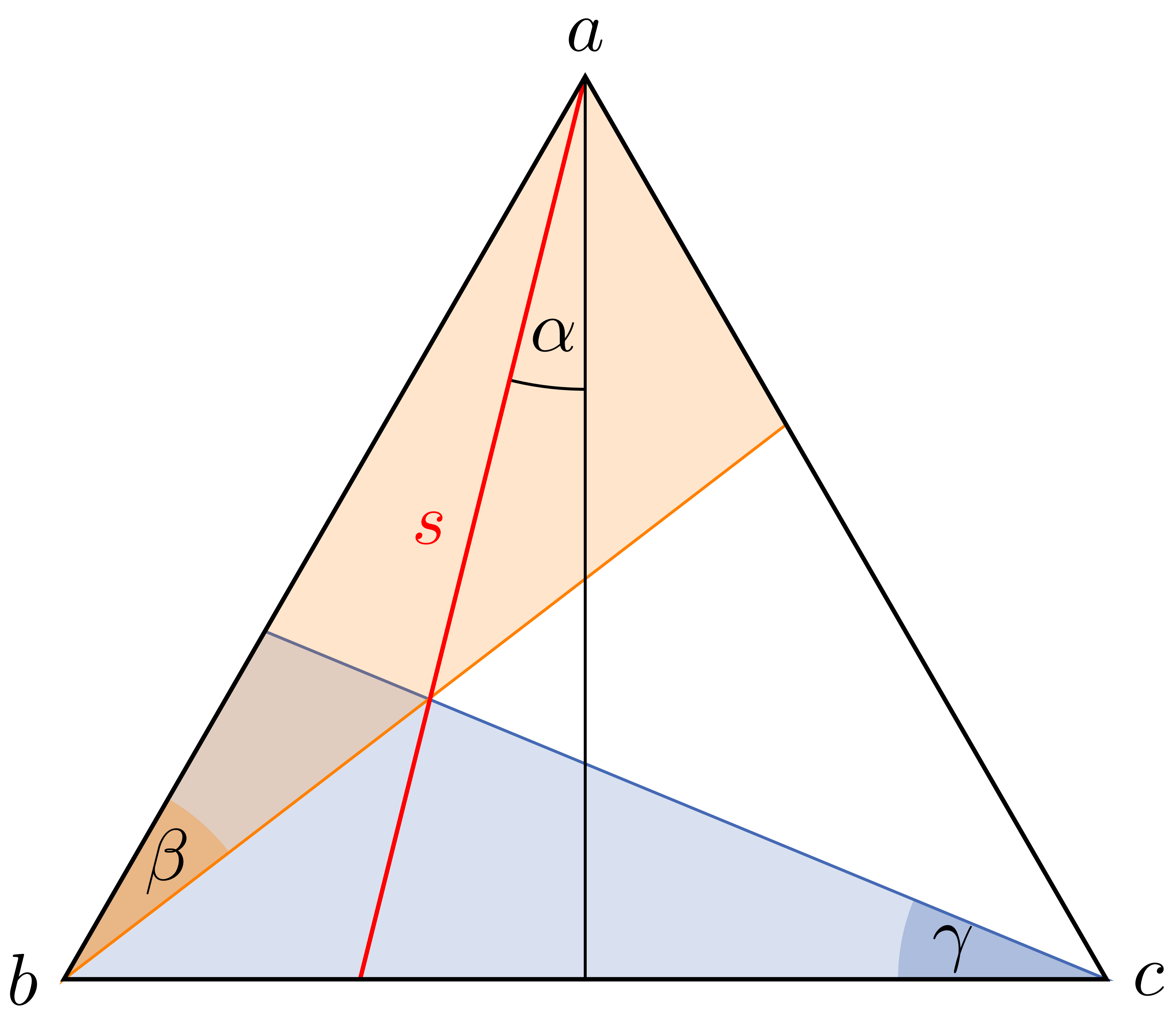

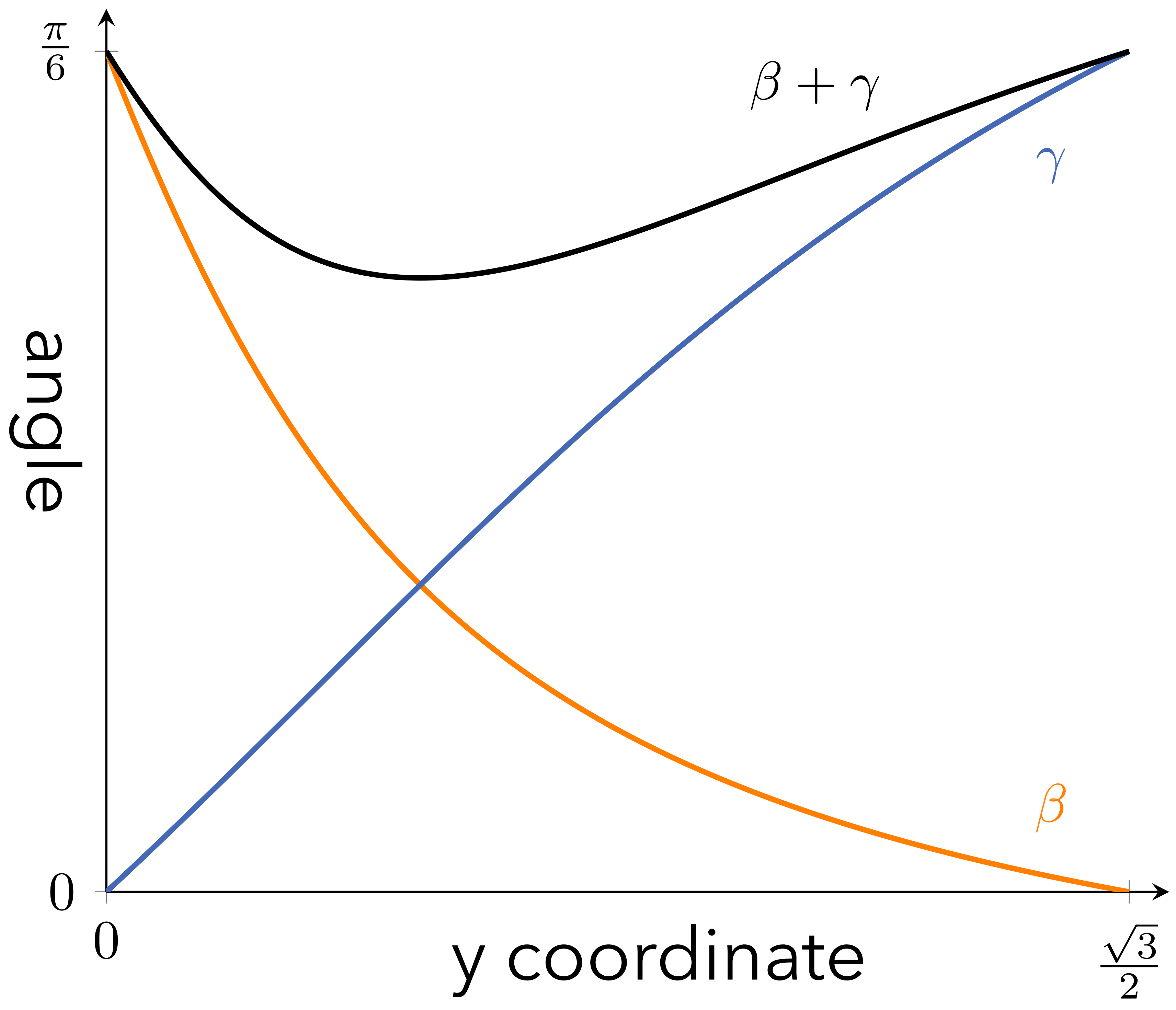

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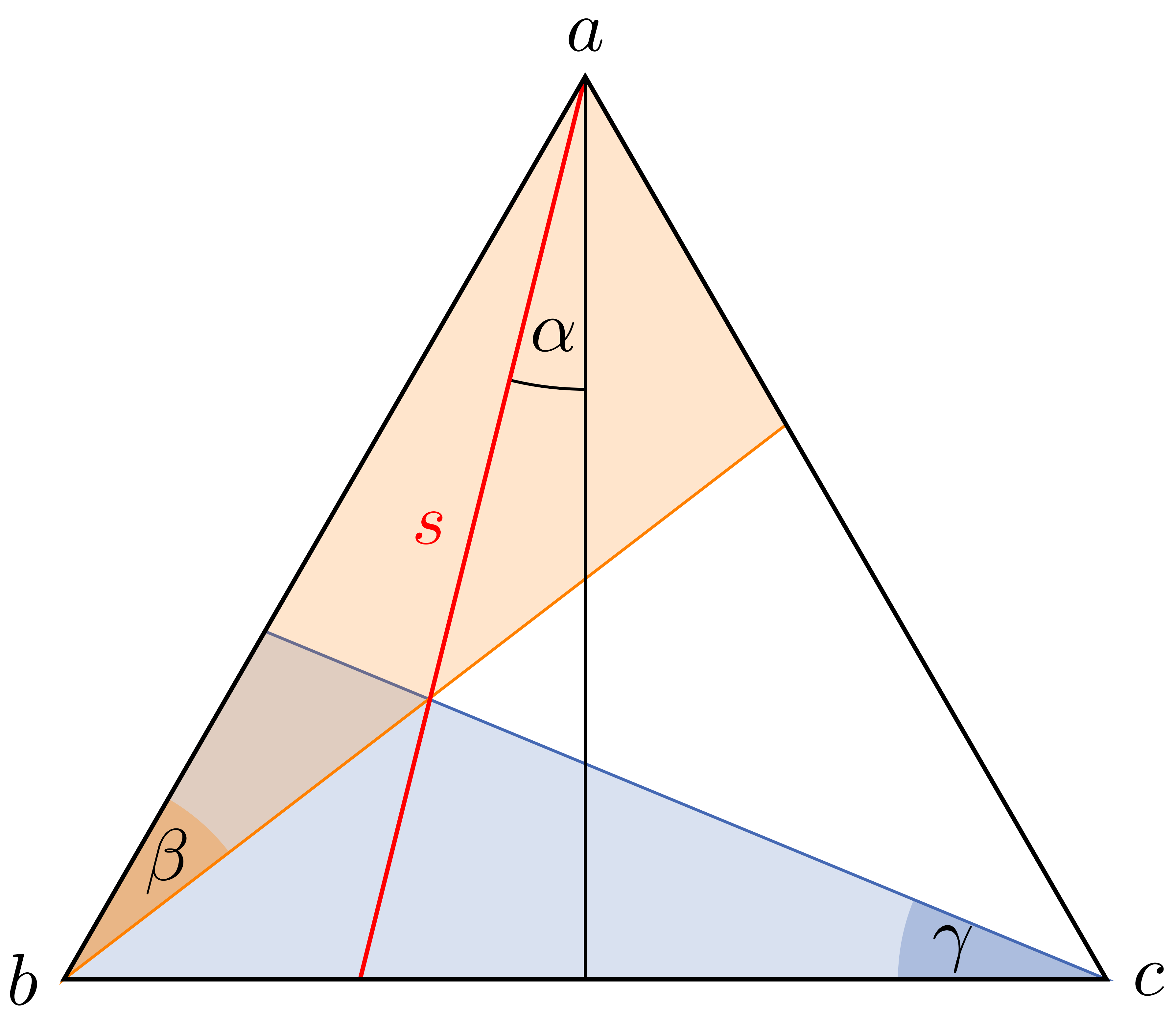

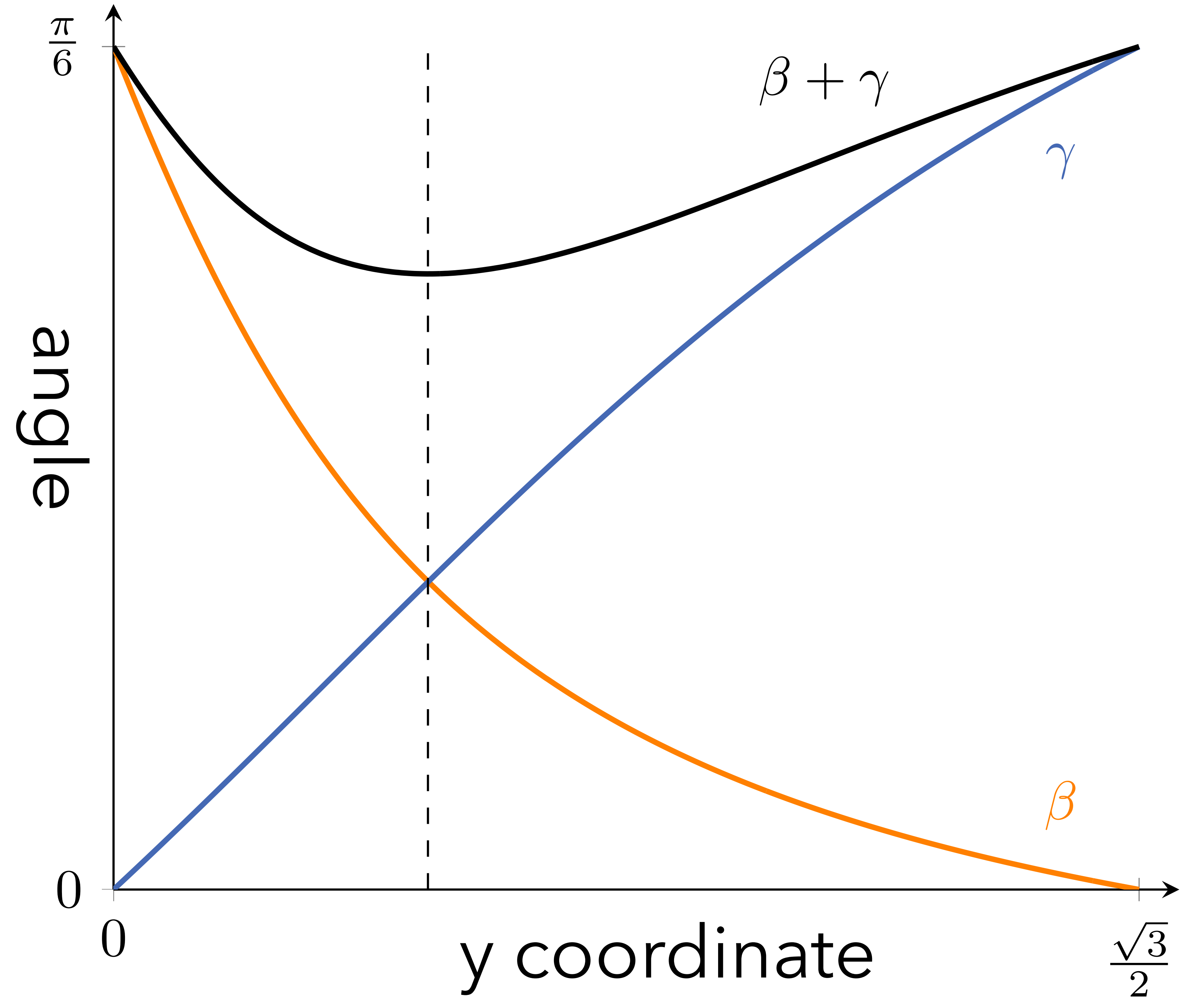

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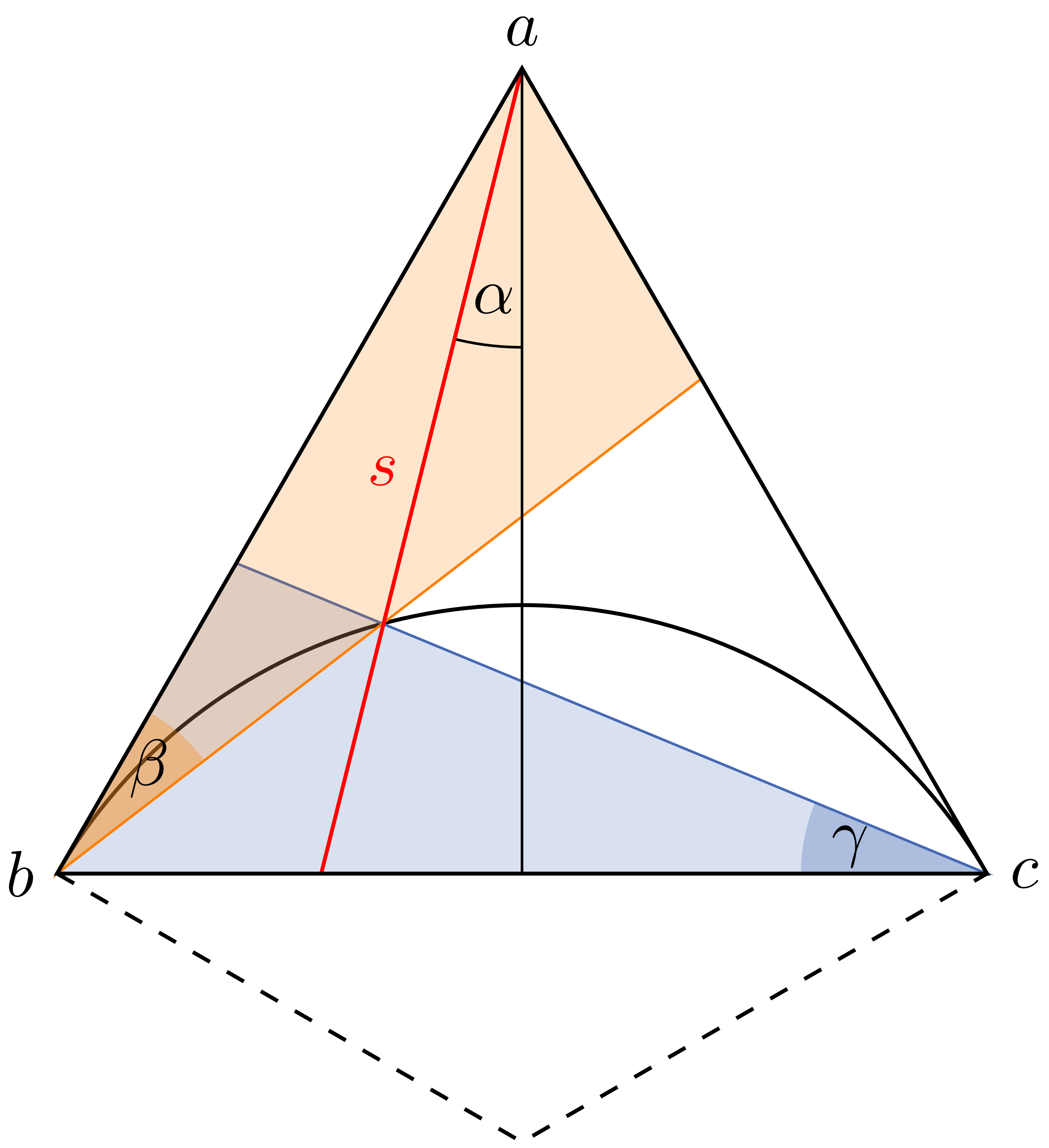

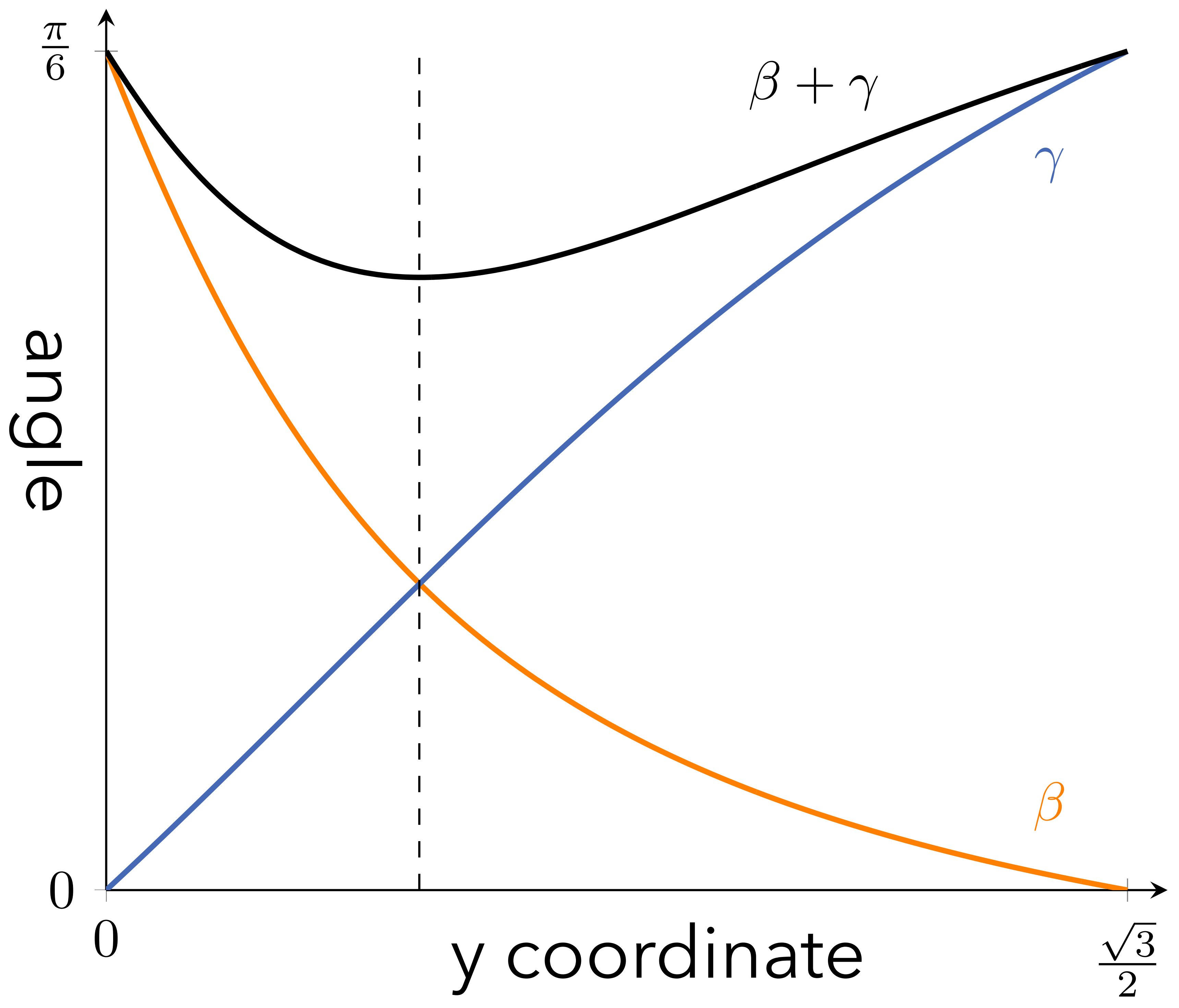

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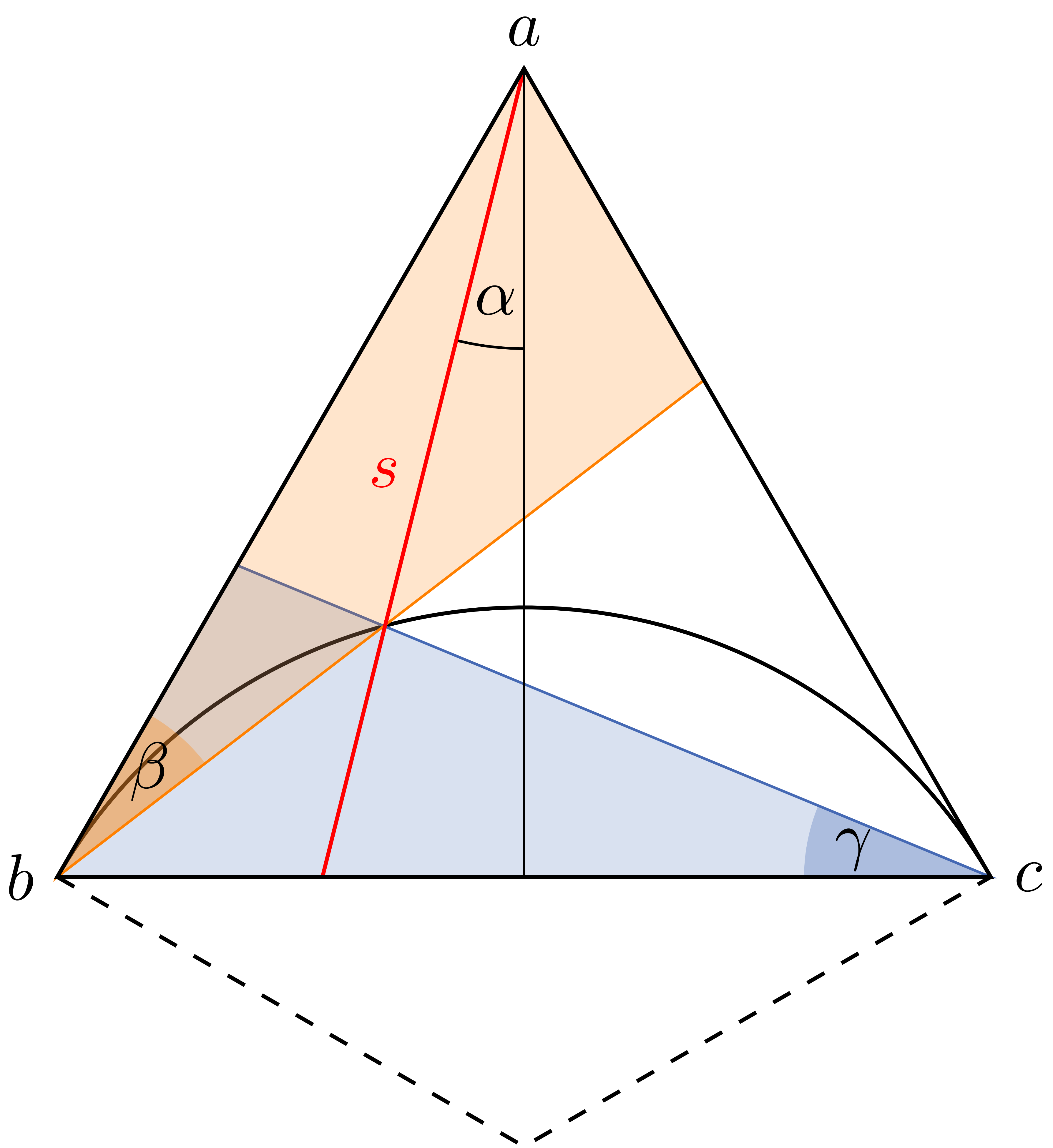

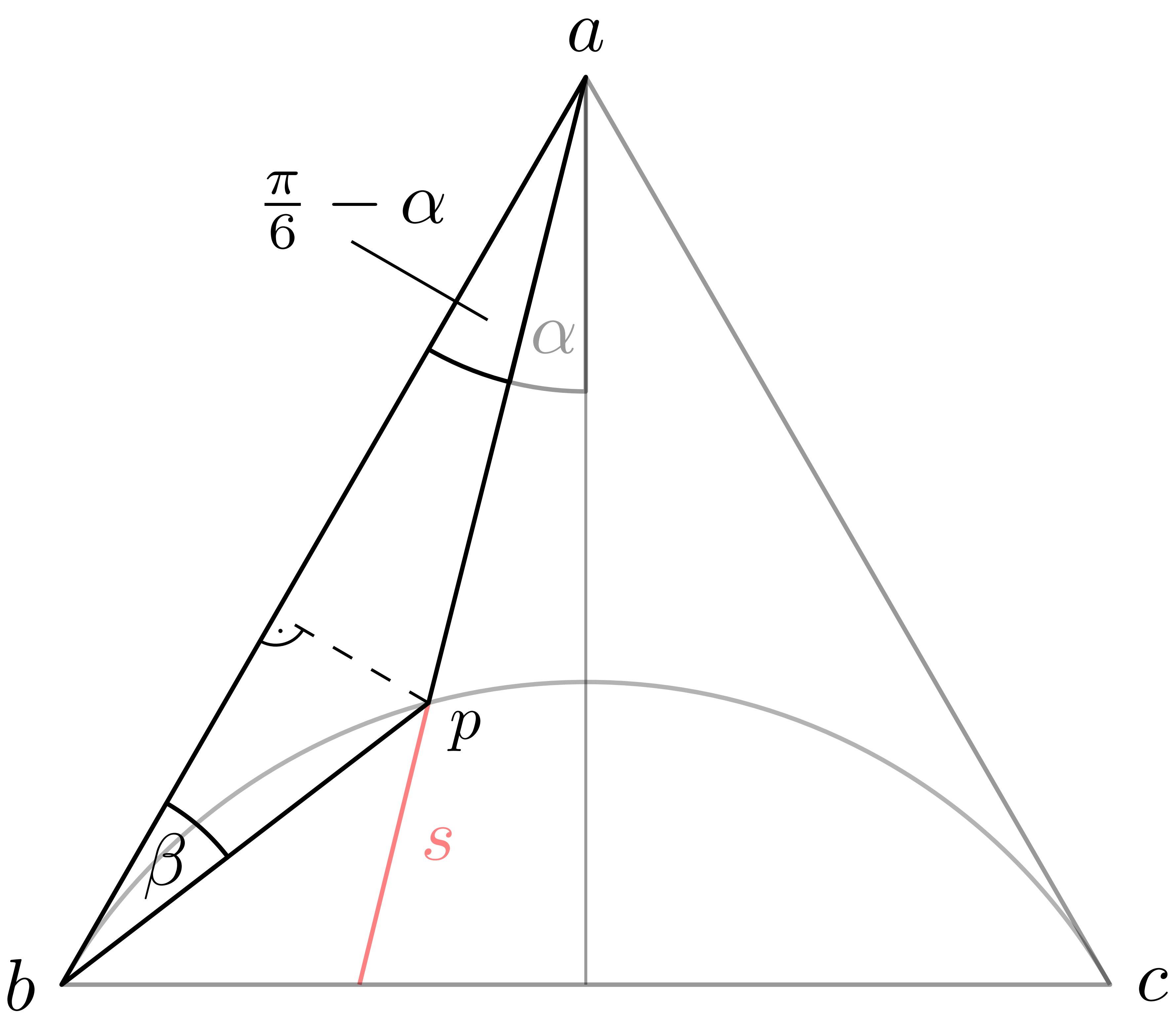

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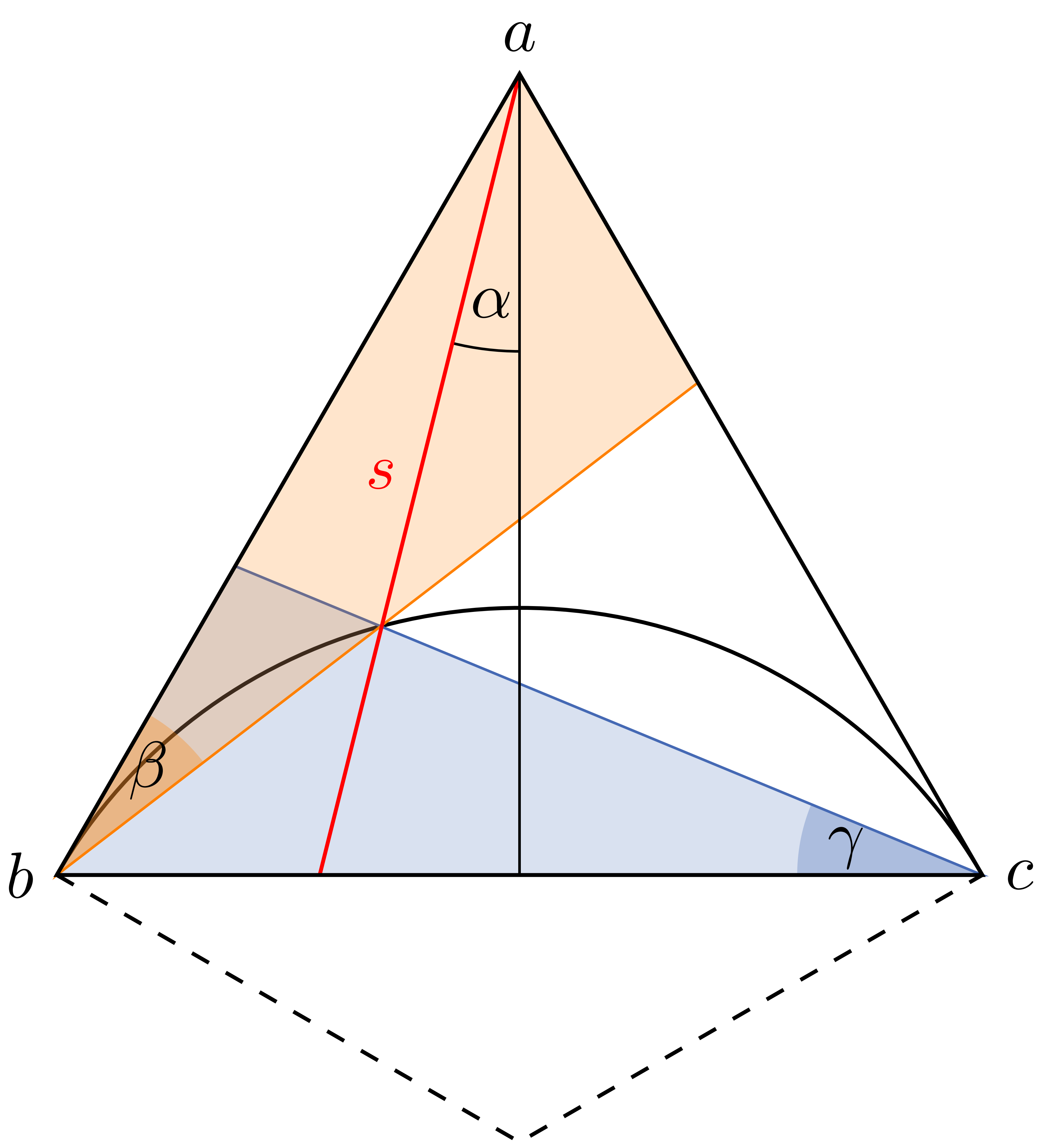
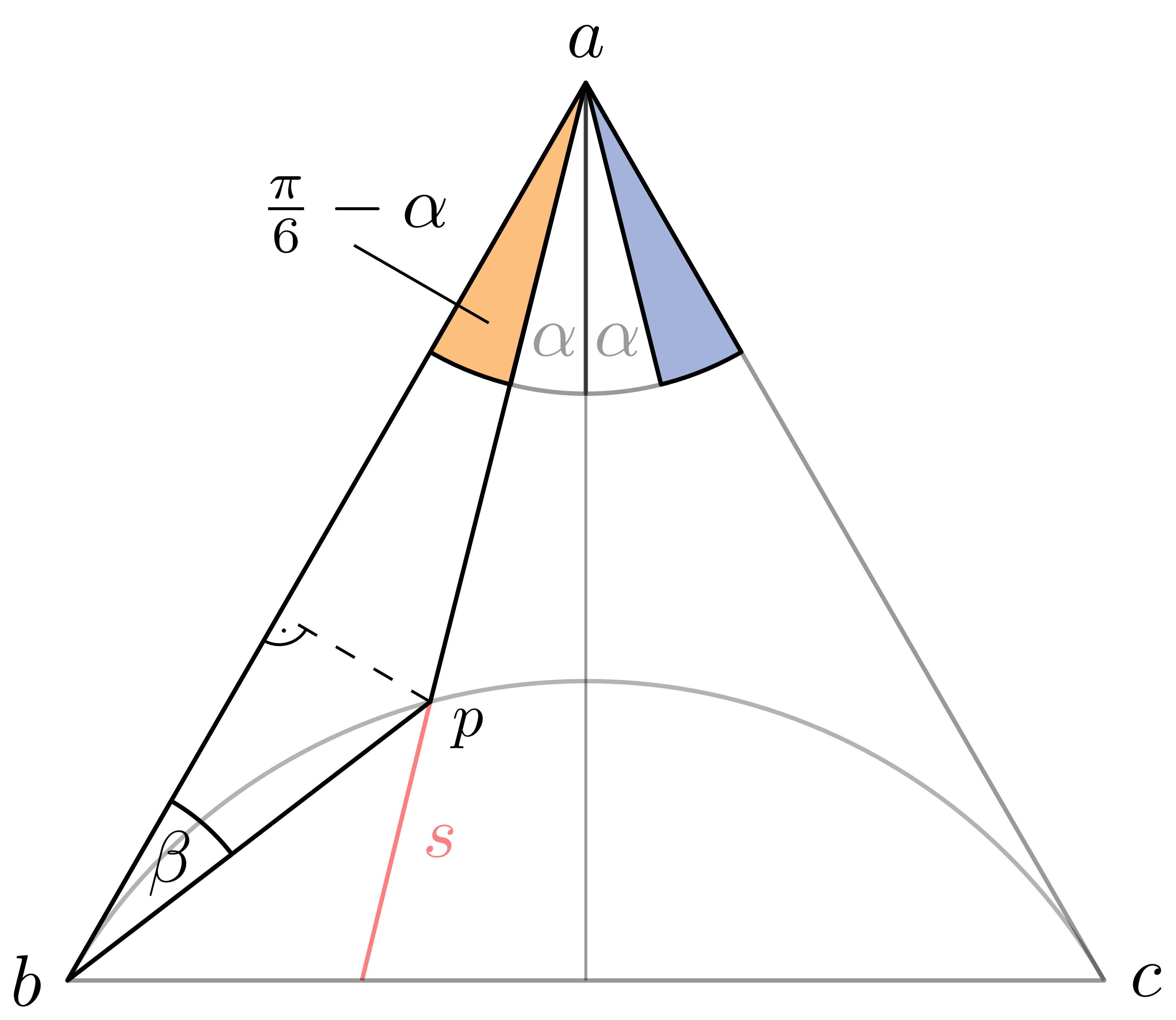
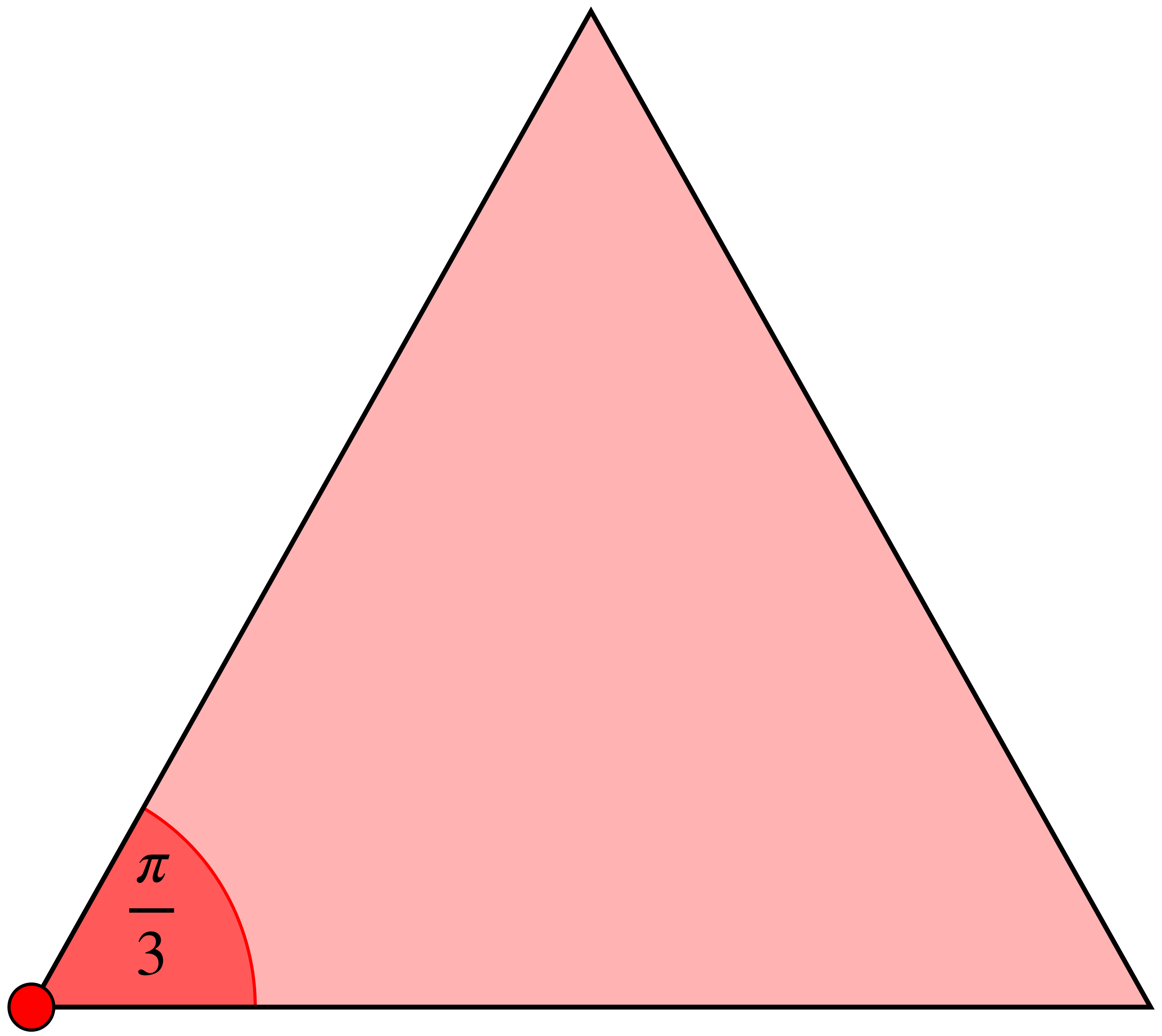




Optimal Covering of Equilateral Triangles



Introduction

Equilateral Triangles

Tight Upper Bound for **Histograms**

Simple Polygons

Duality

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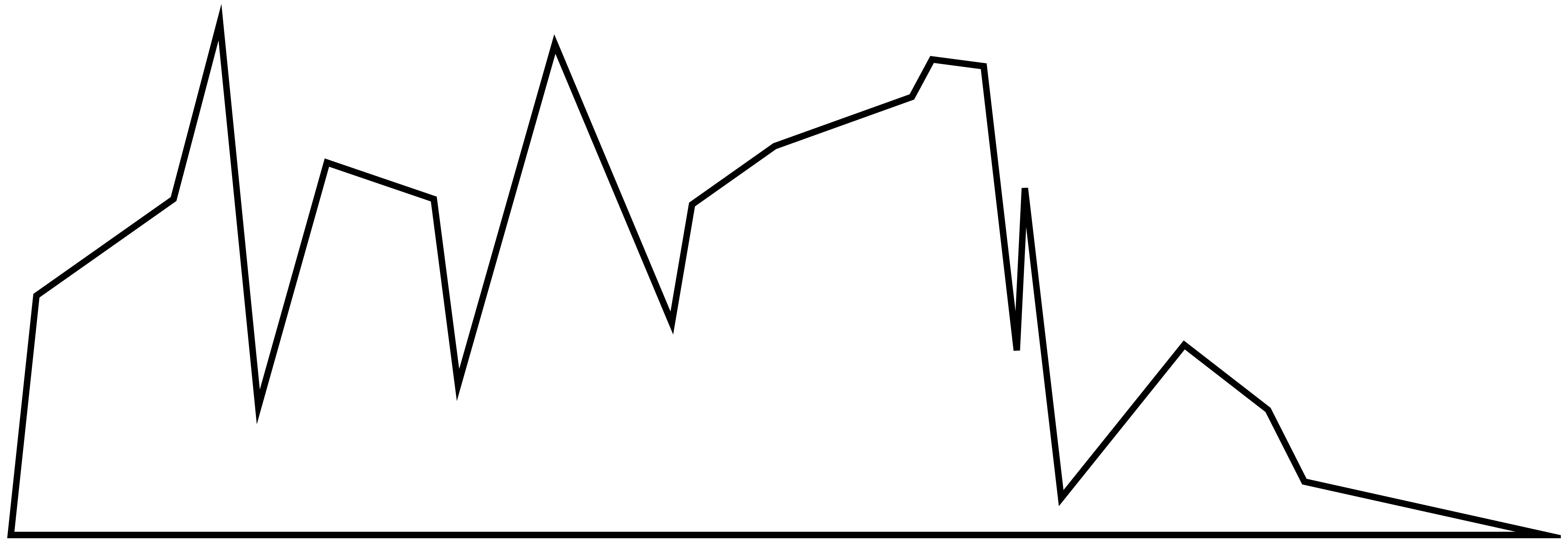

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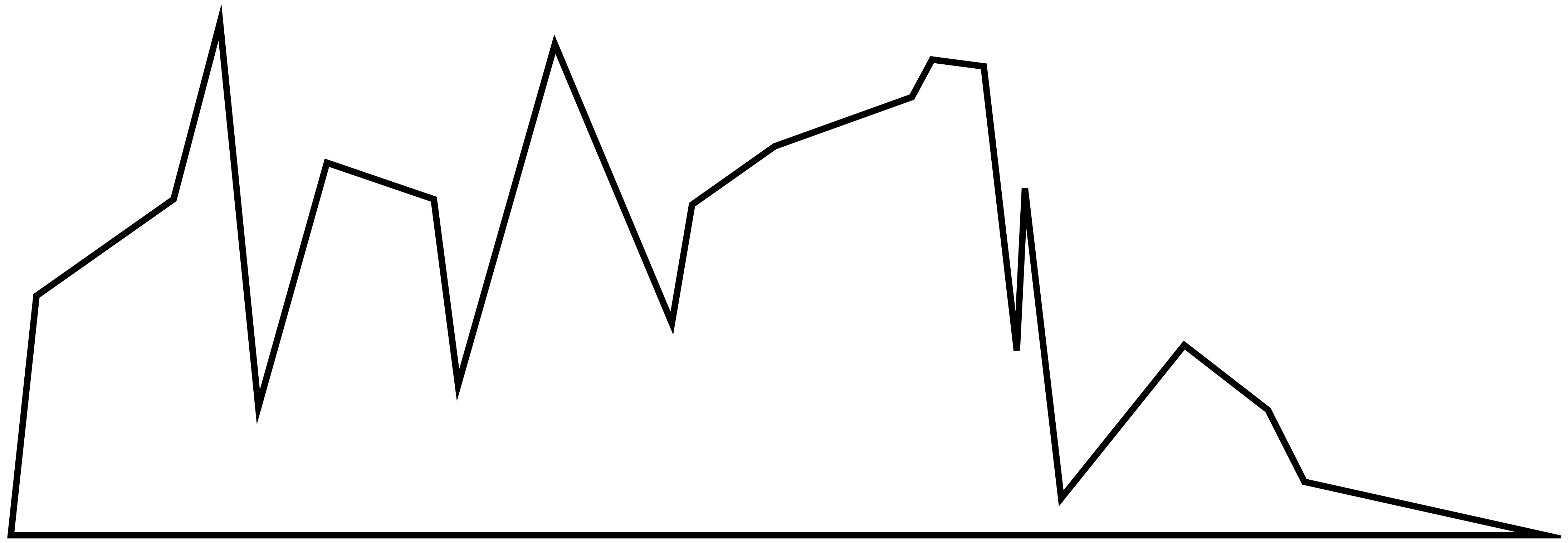

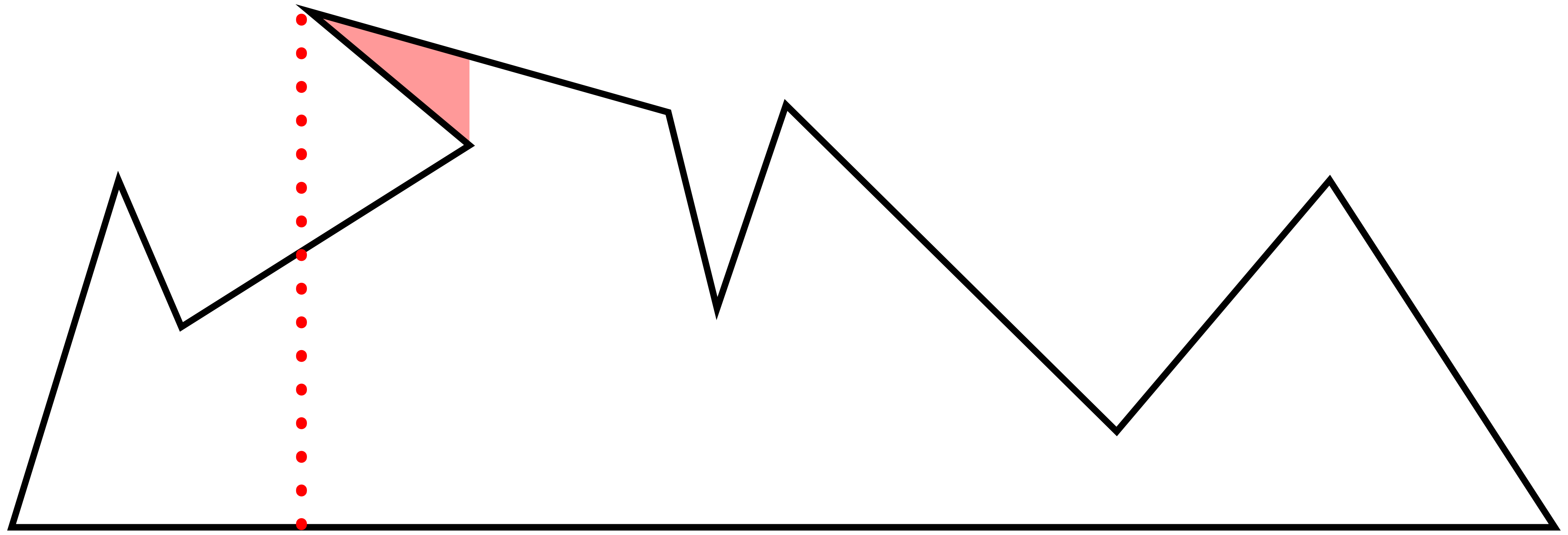

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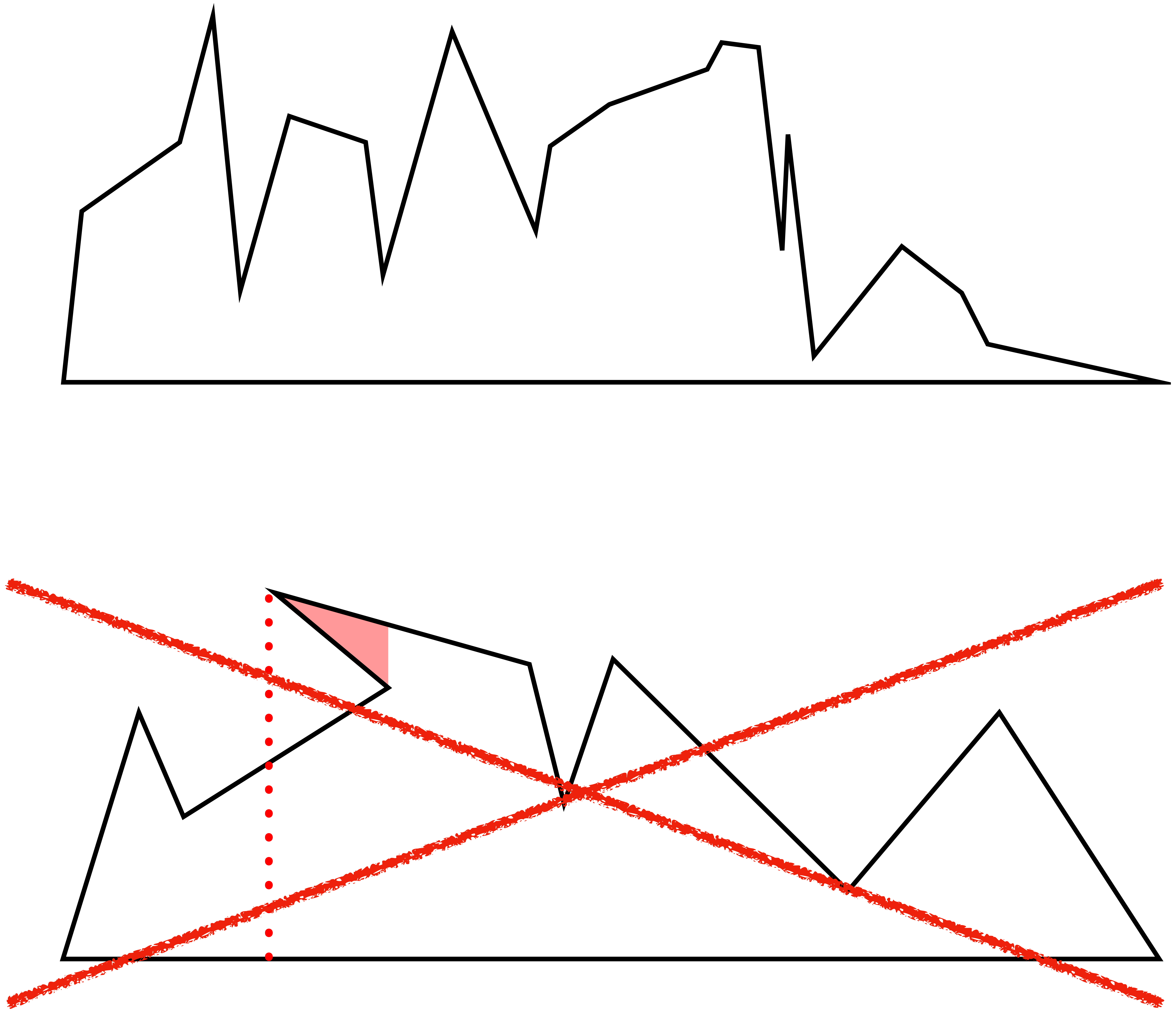

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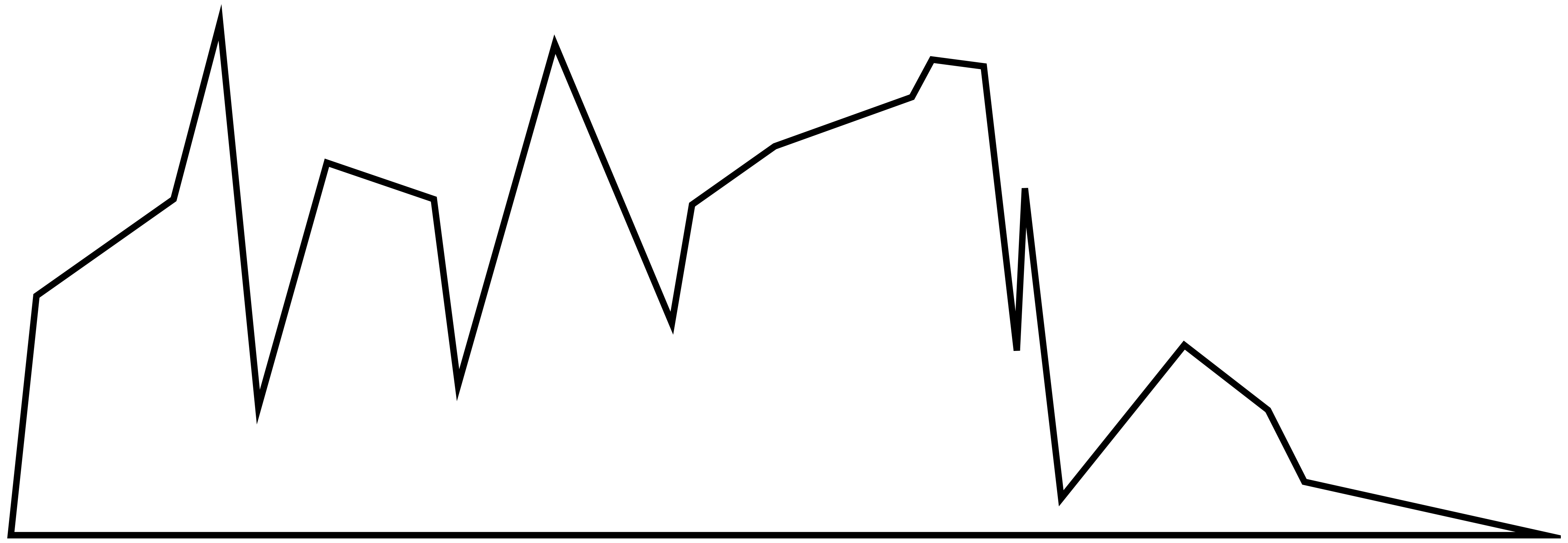
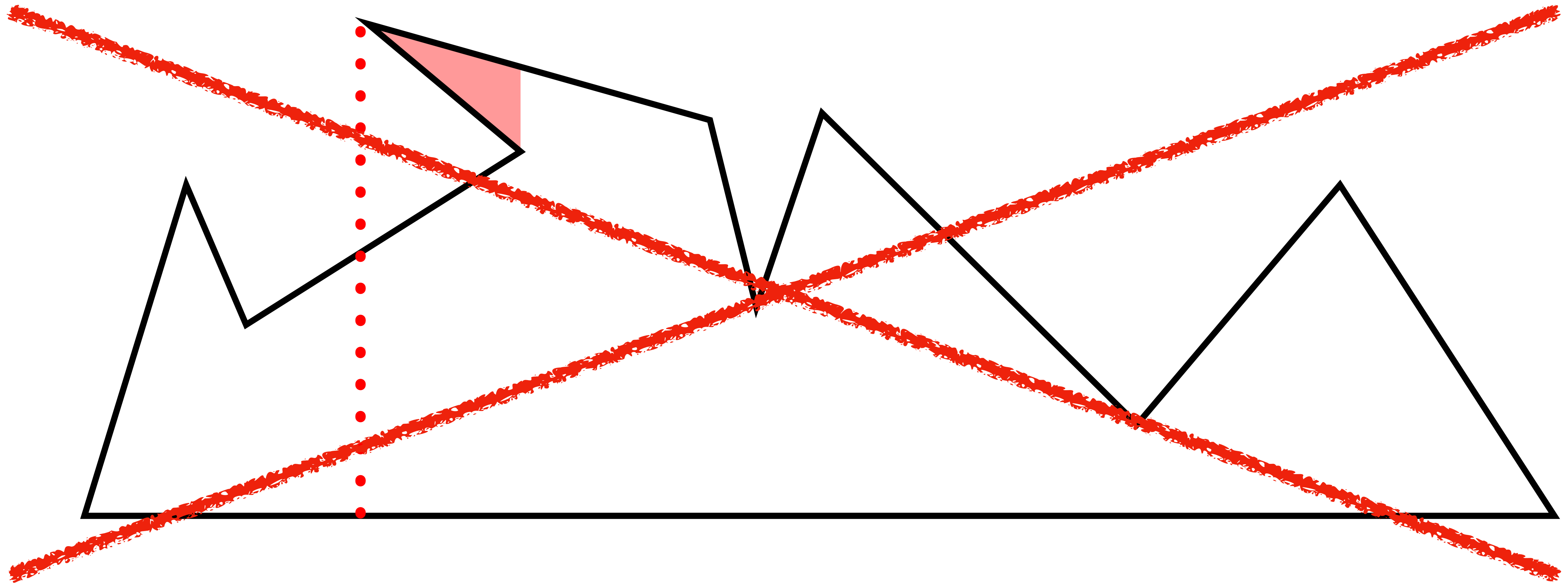
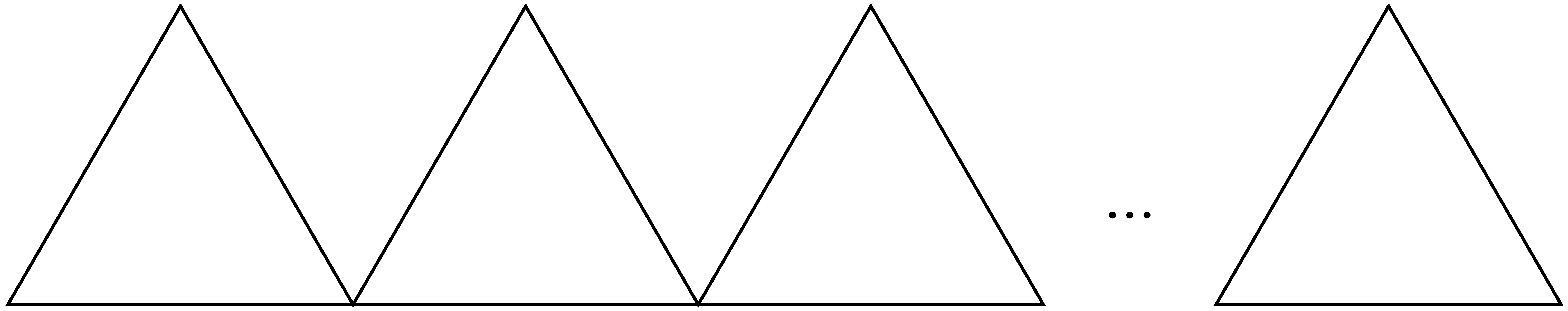


$$(n-1)\frac{\pi}{6}$$



Lower Bound for Histograms



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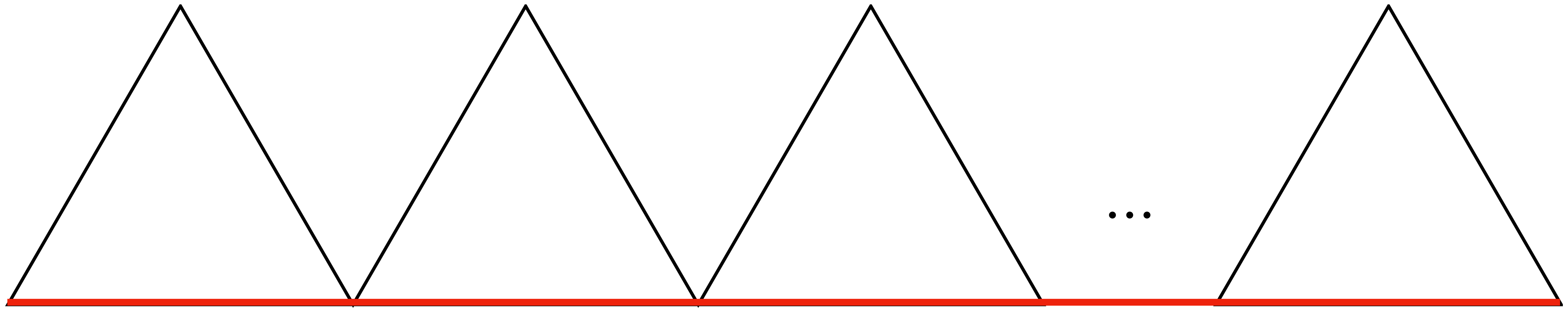

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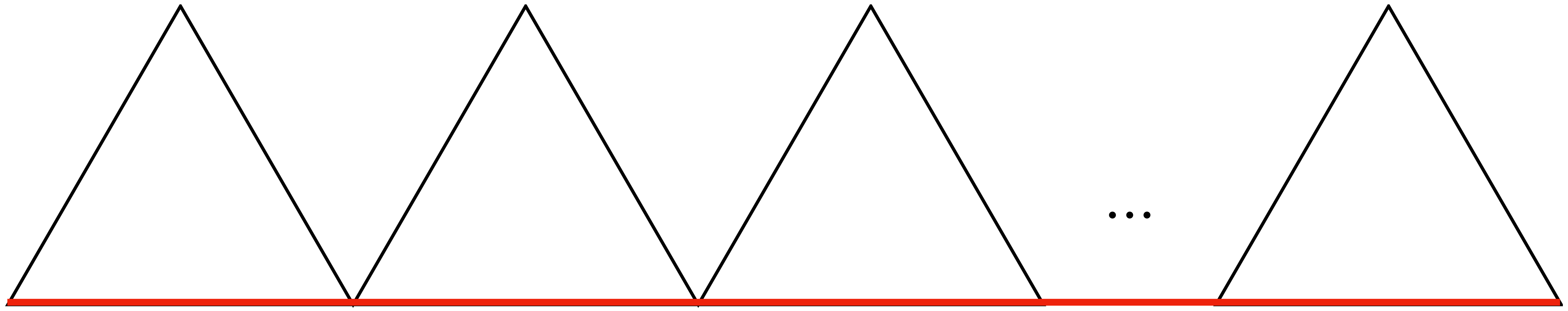
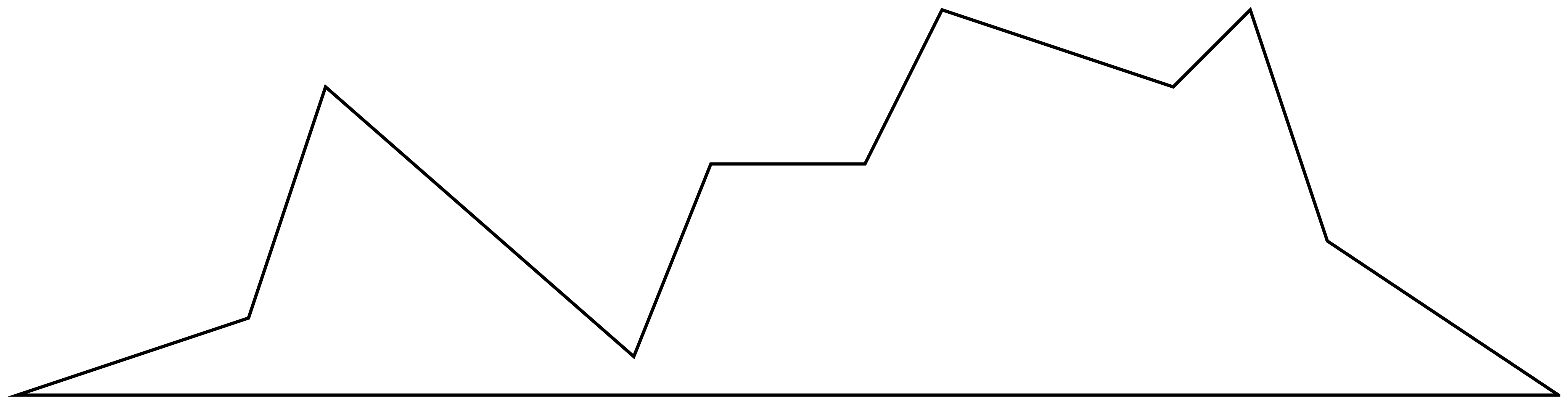
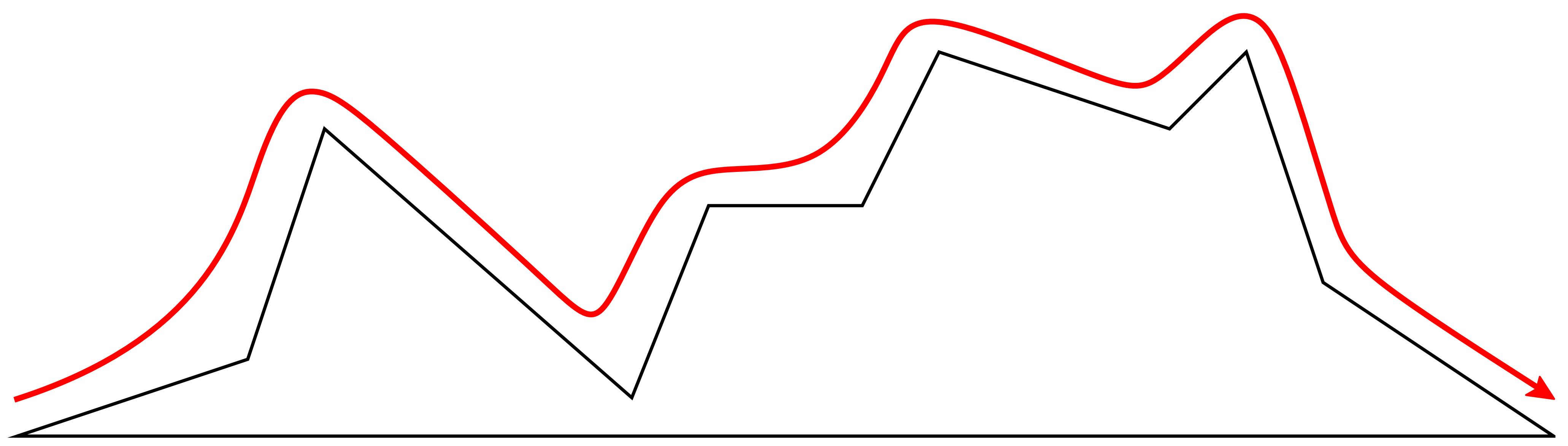


Lower Bound: $$\frac{n-1}{2}\frac{\pi}{3} = (n-1)\frac{\pi}{6}$$

Upper Bound for Histograms



Upper Bound for Histograms



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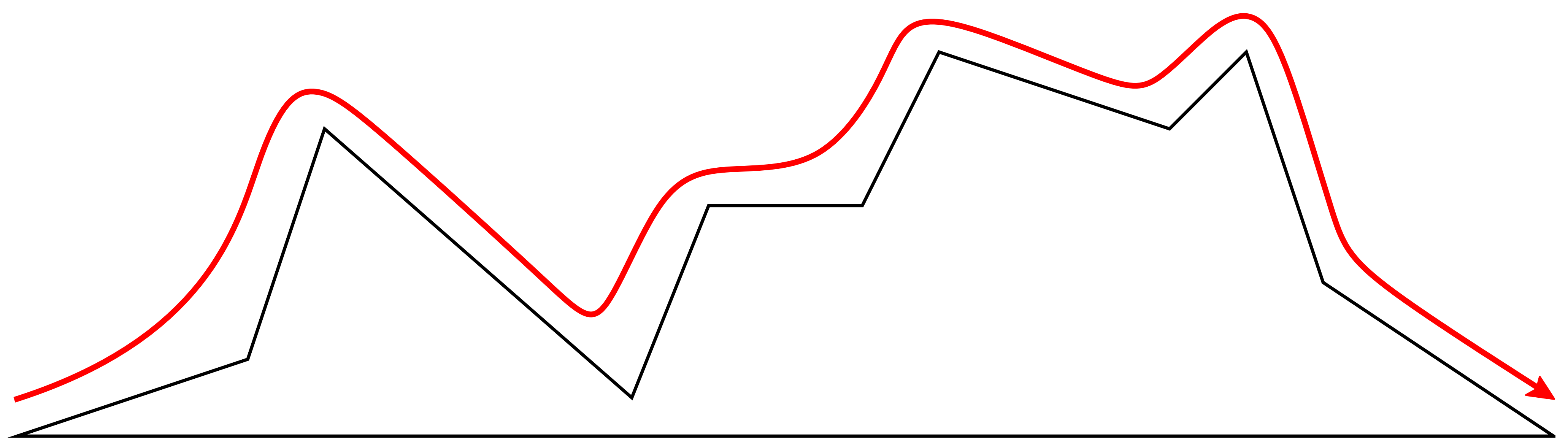

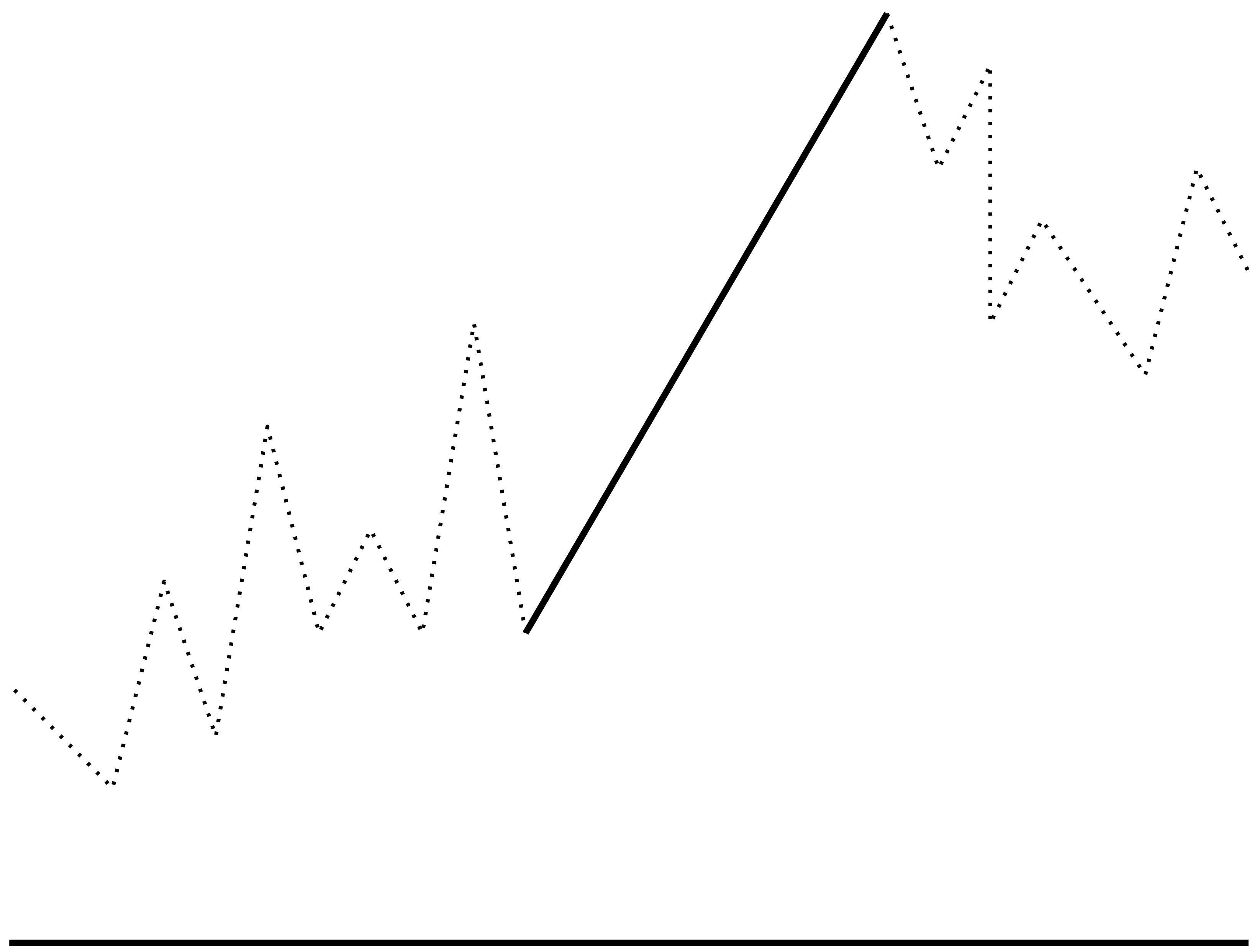

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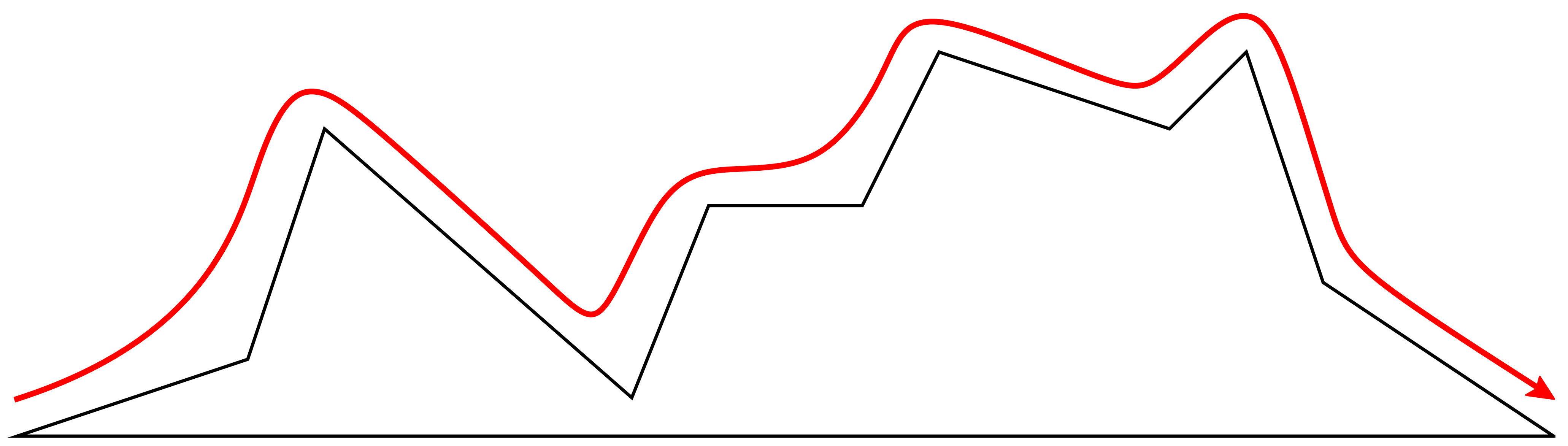
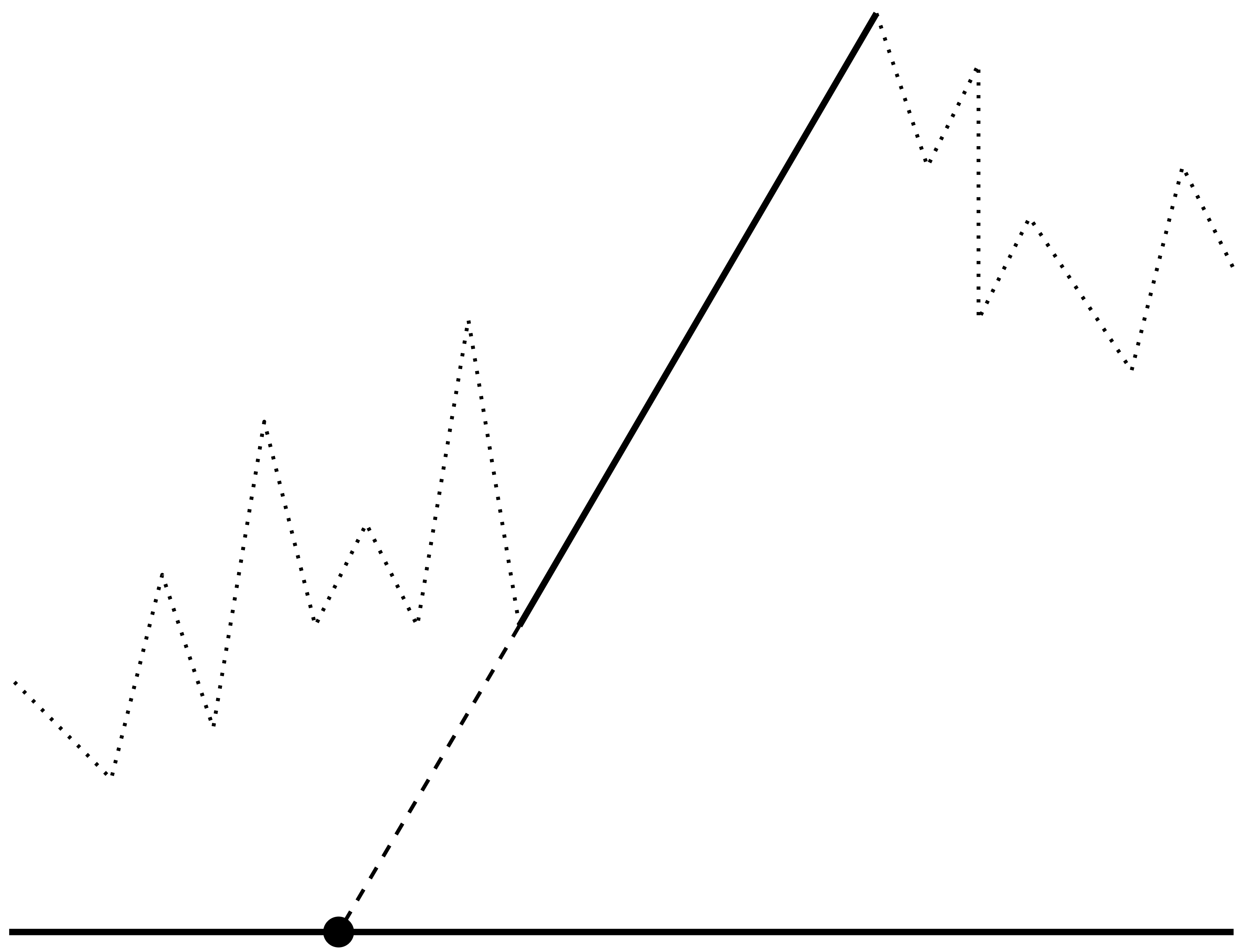
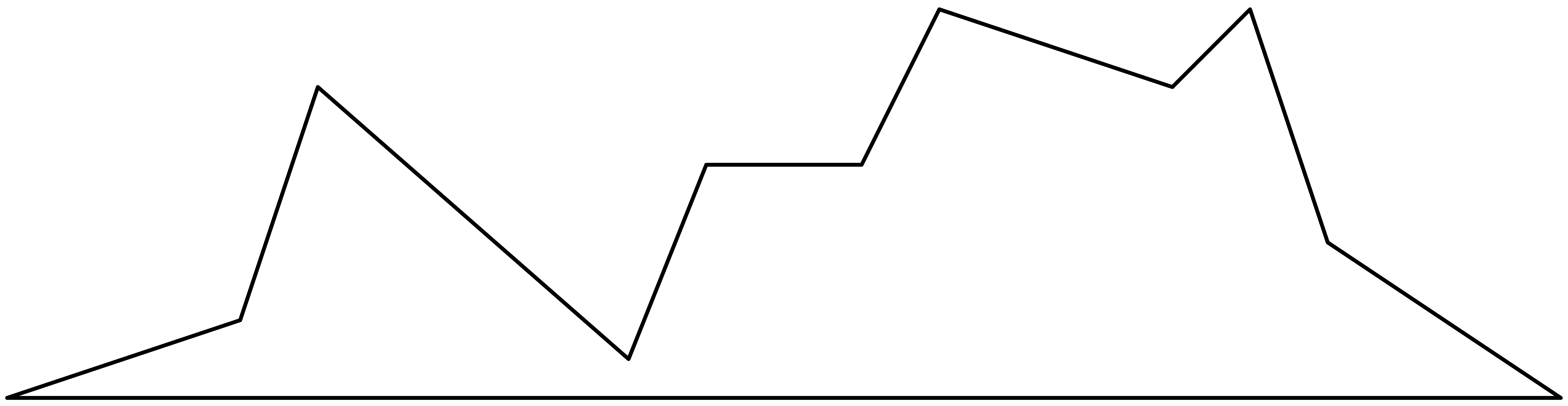




Histogram Covering Algorithm – Example



Case	Condition
1	e_i is covered
2	v_{i+1} is reflex
3	v_{i+2} is convex
4	there is another uncovered, completely visible edge
5	visibility extension of e_{i+1} intersects with baseline
6	none of the above applies

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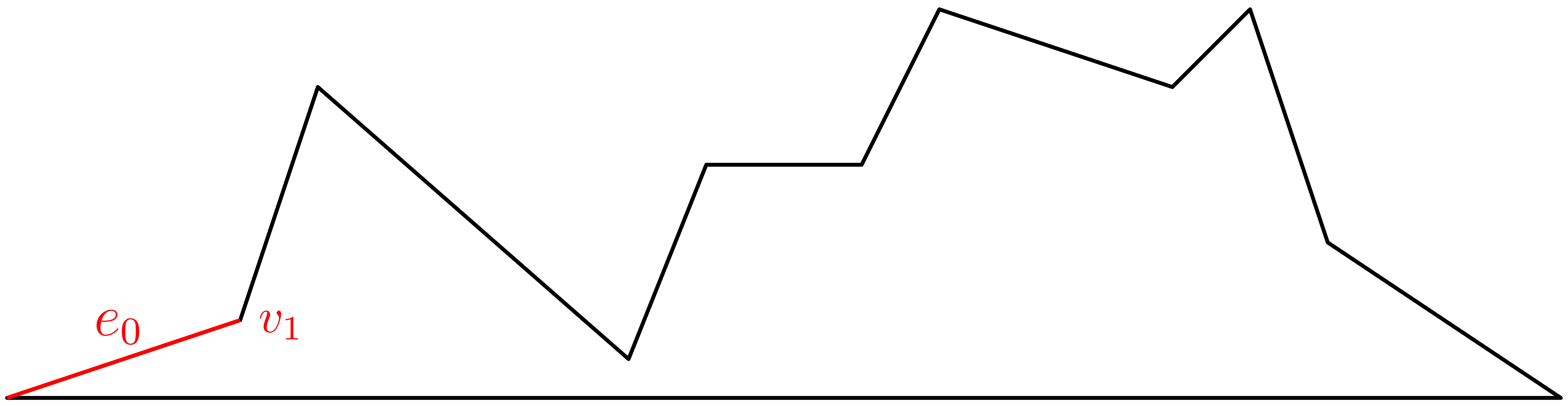

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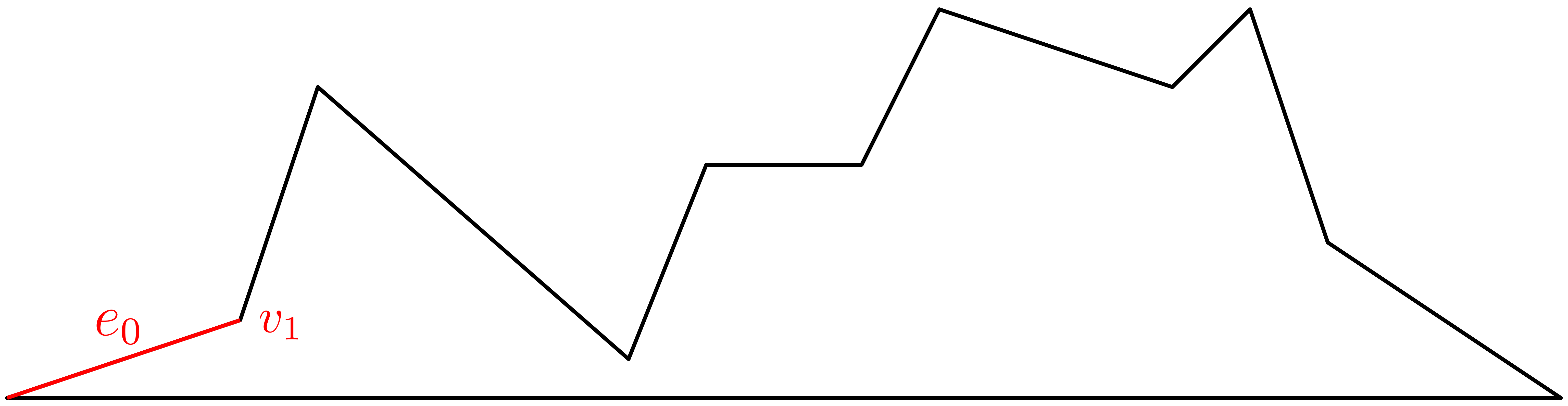

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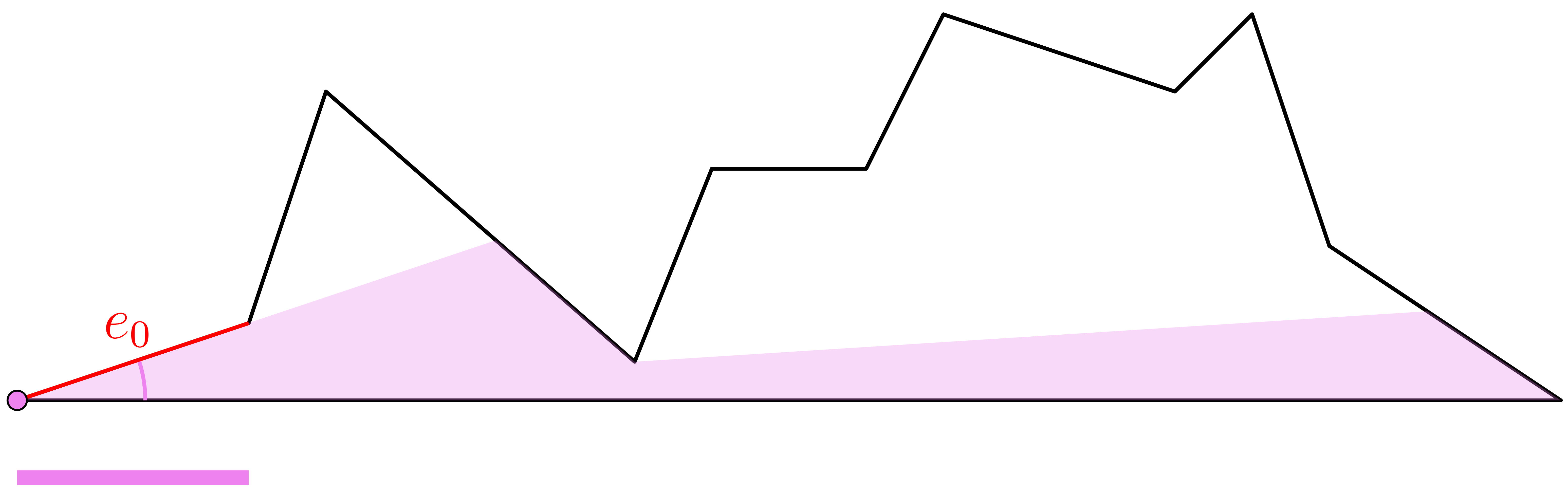

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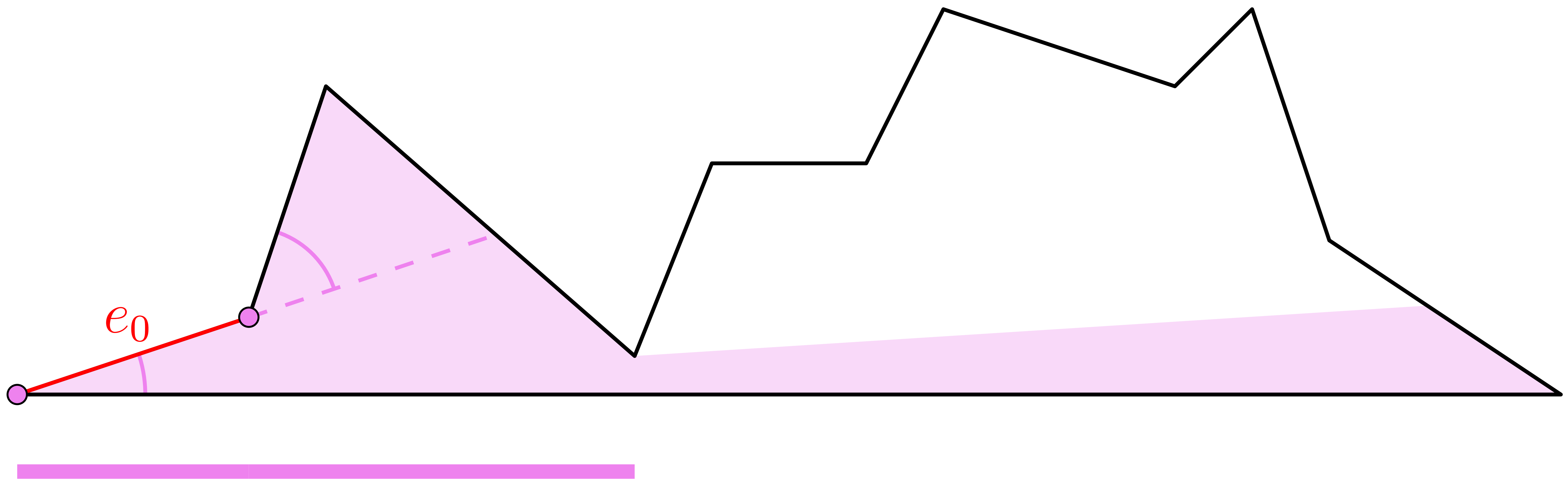

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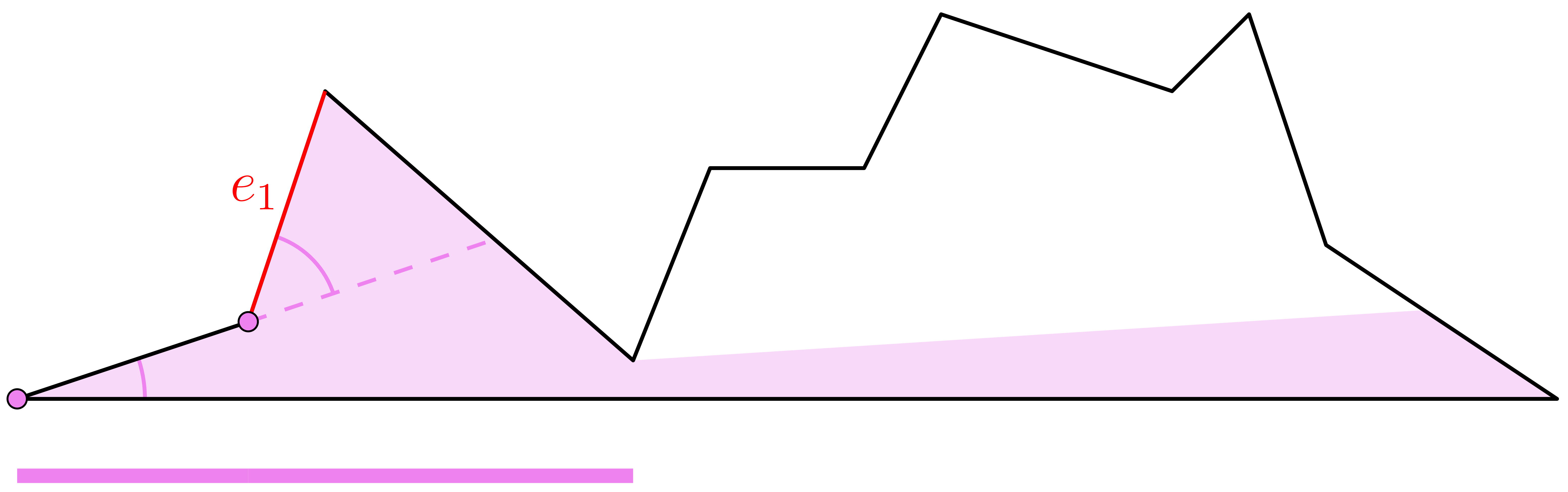

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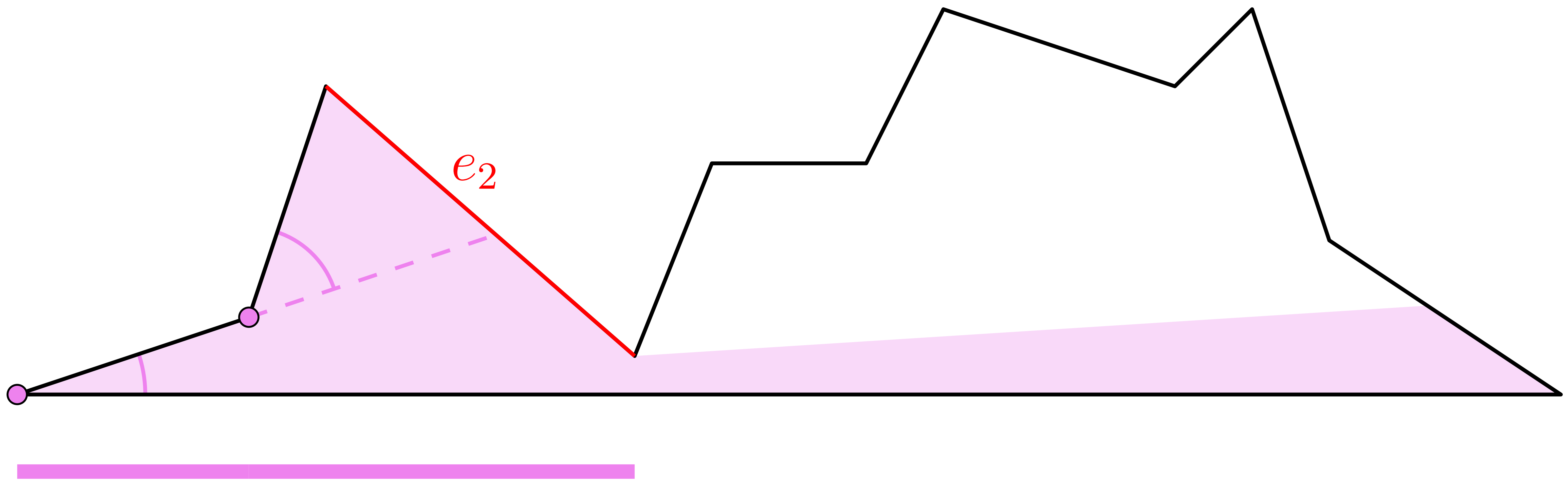

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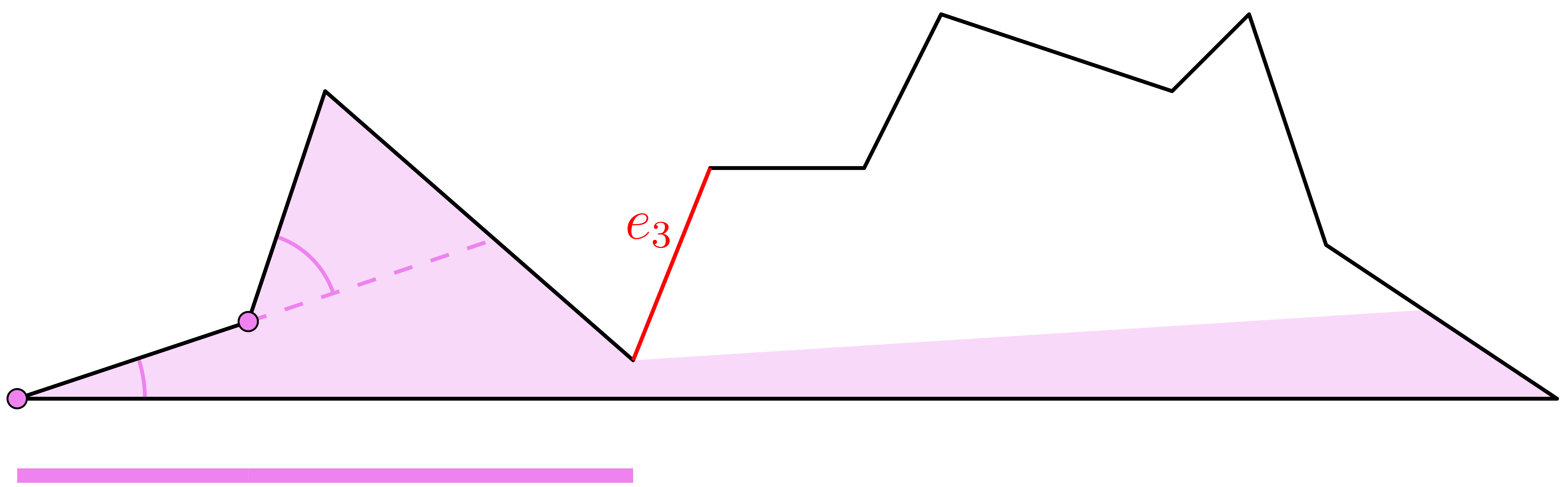

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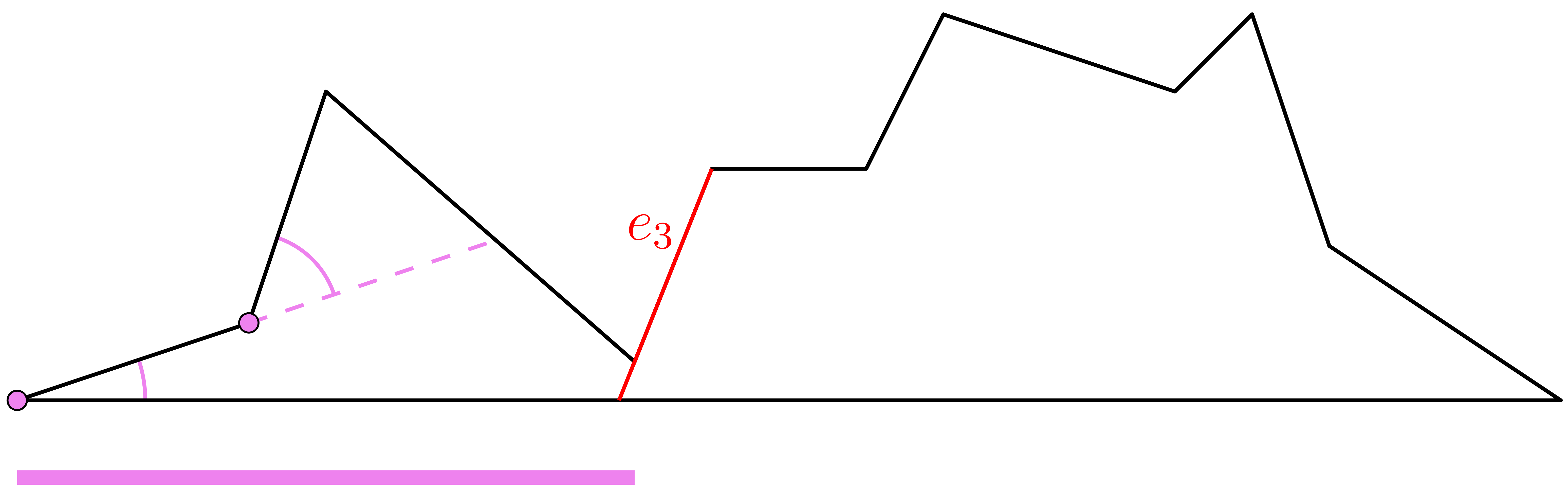
Histogram Covering Algorithm – Example



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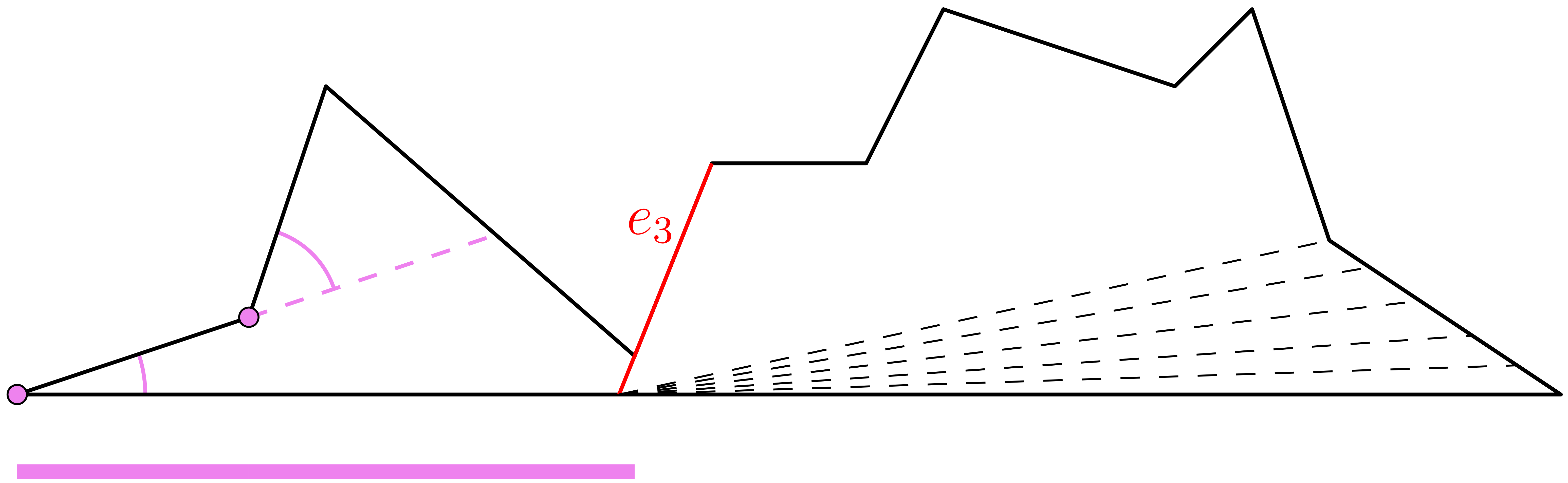

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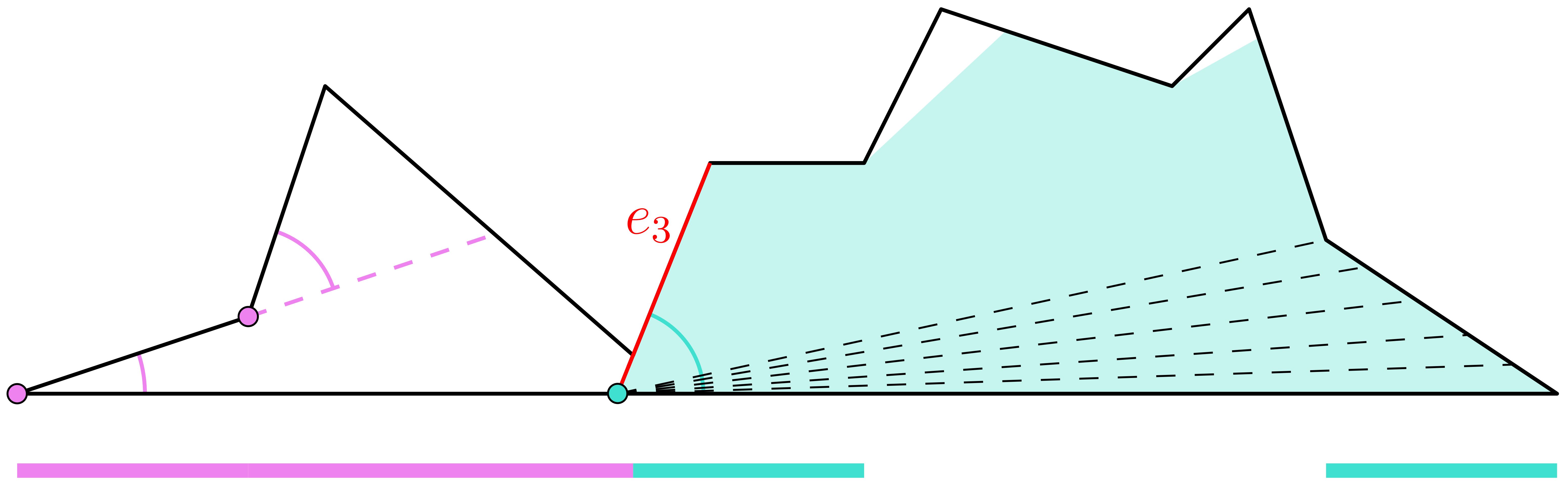

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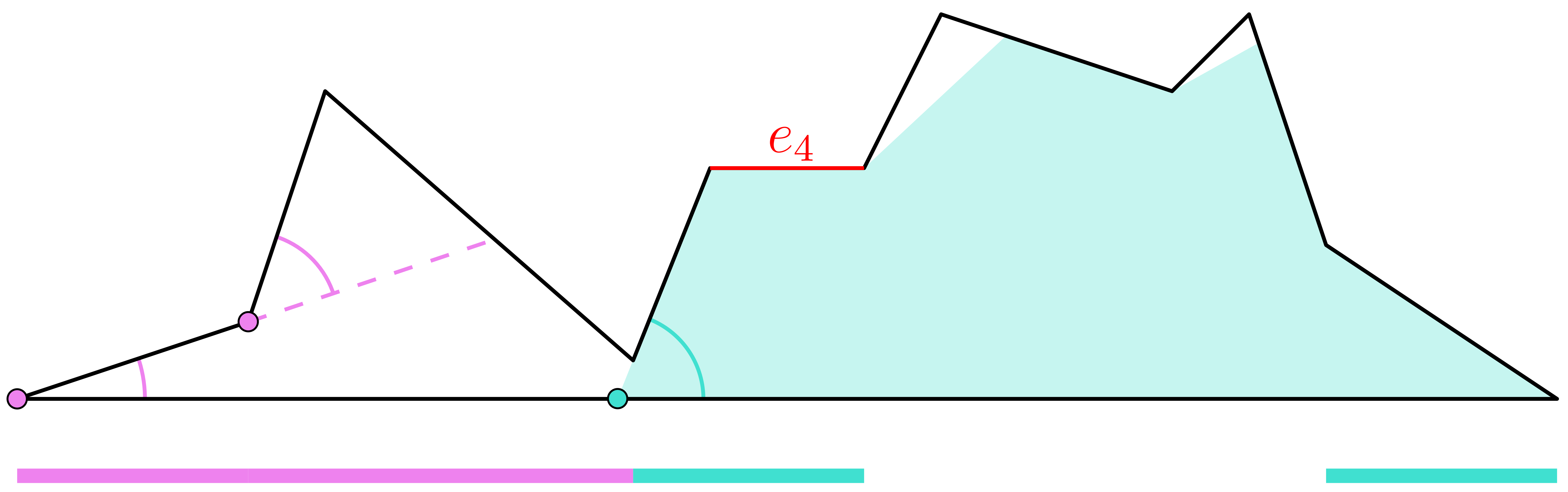

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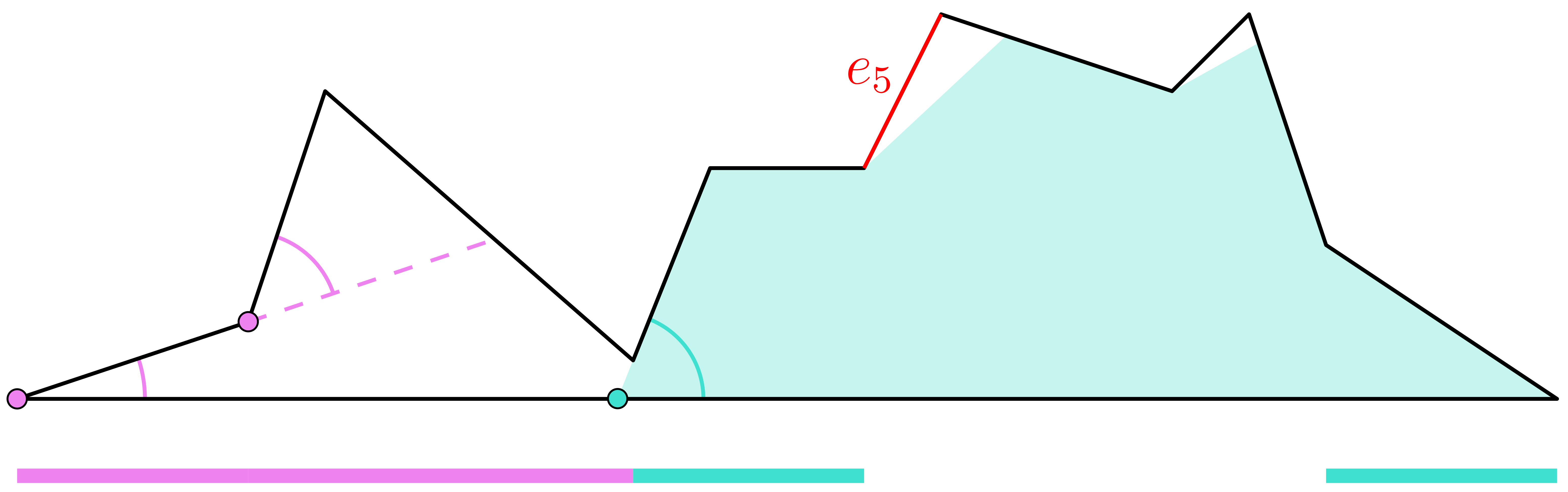

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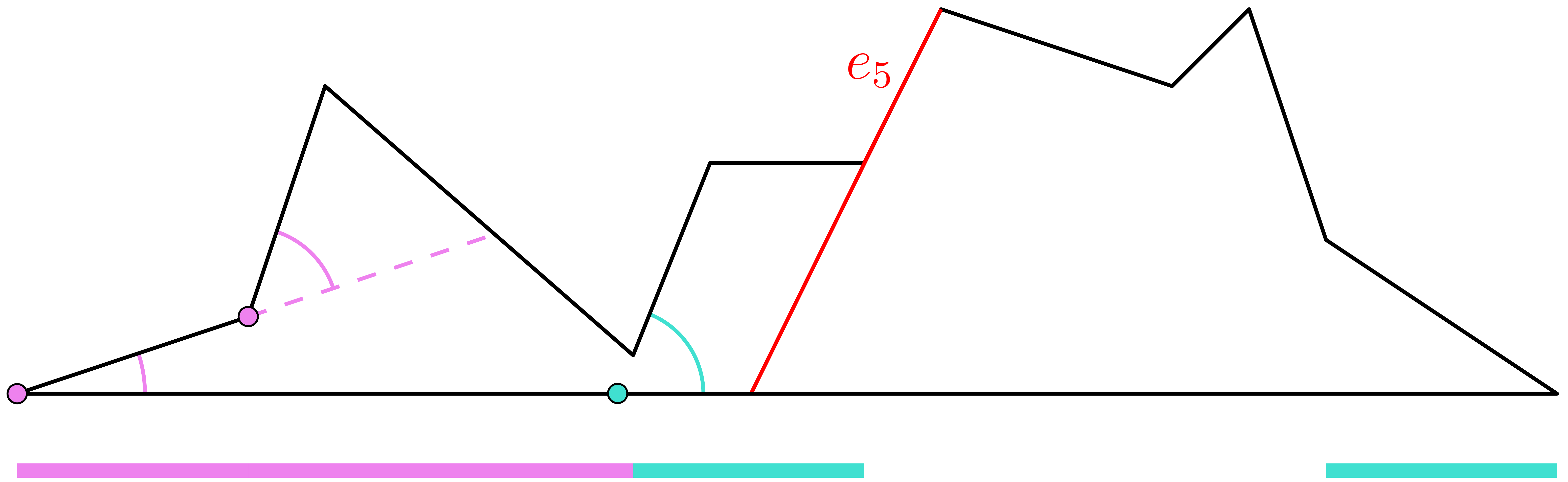

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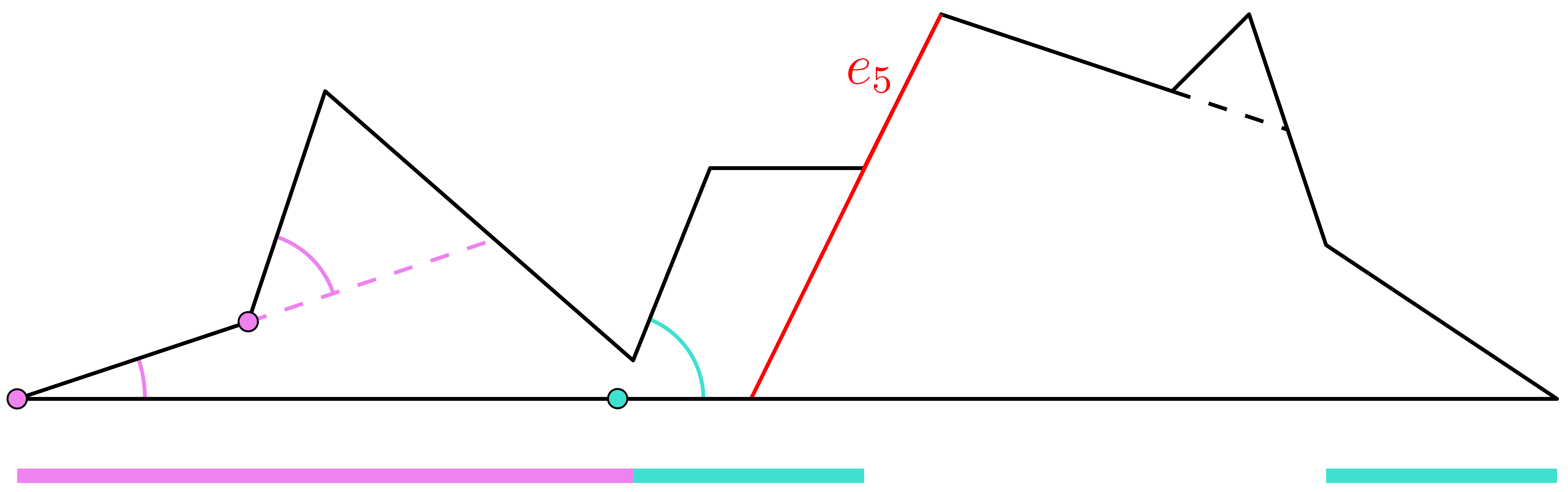

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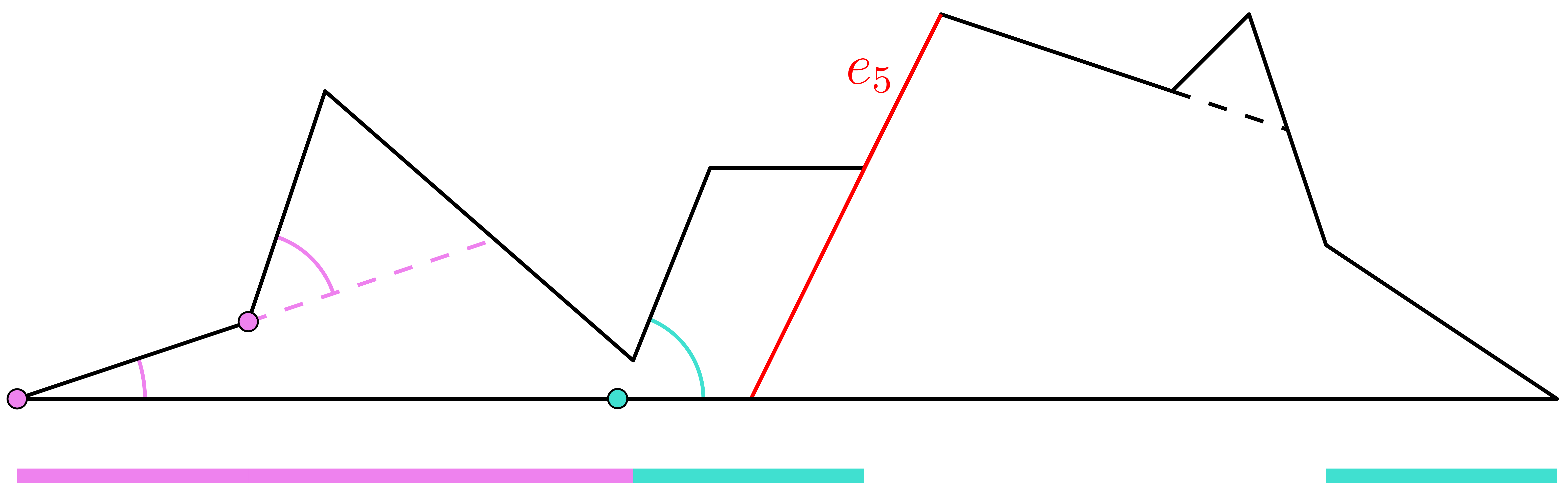

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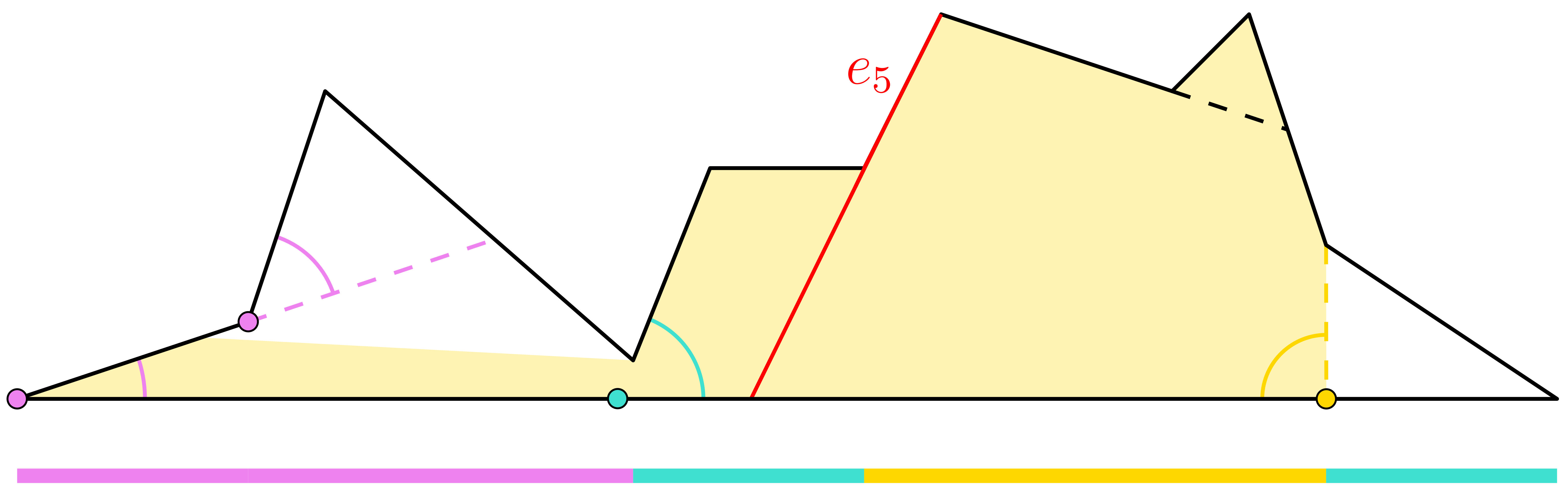

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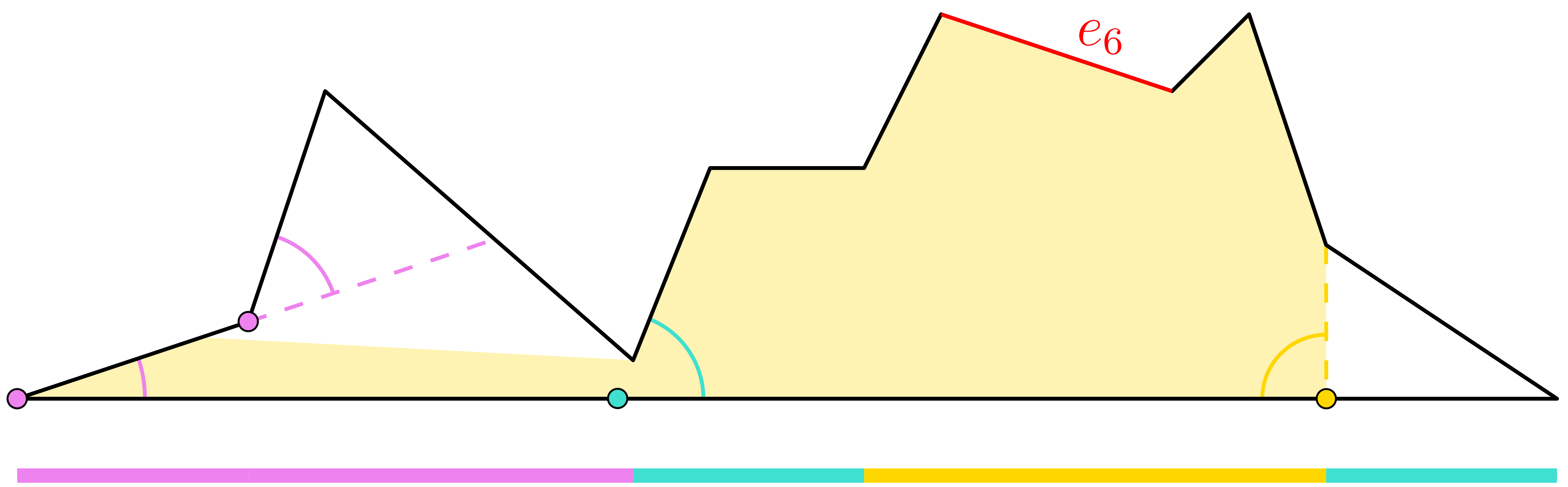

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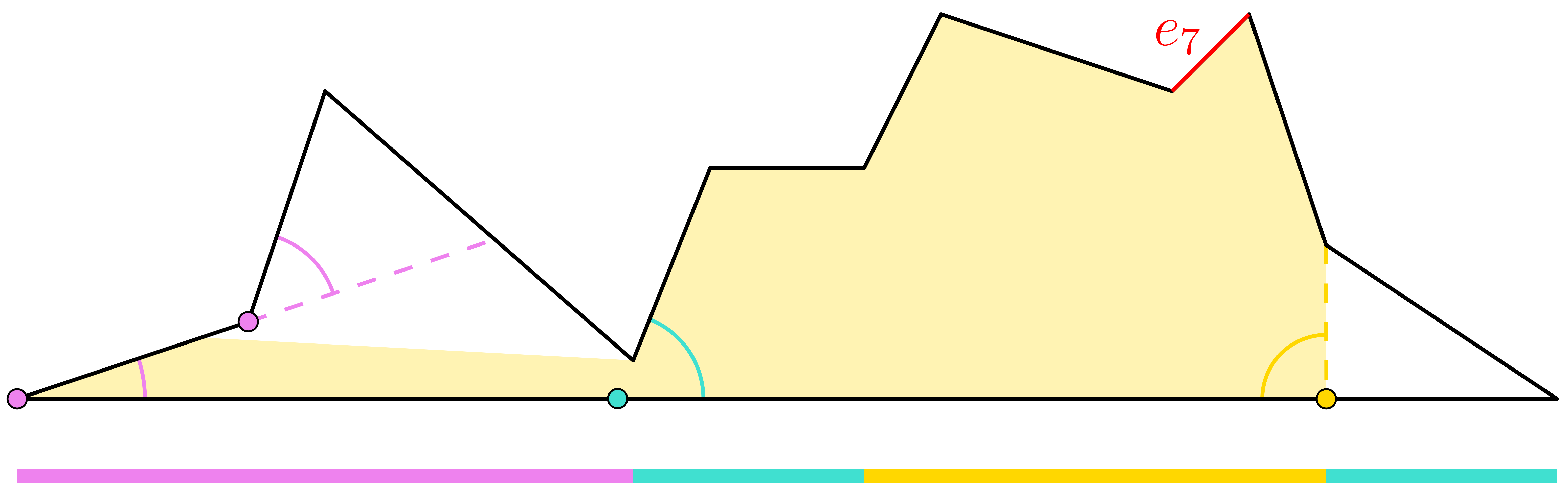

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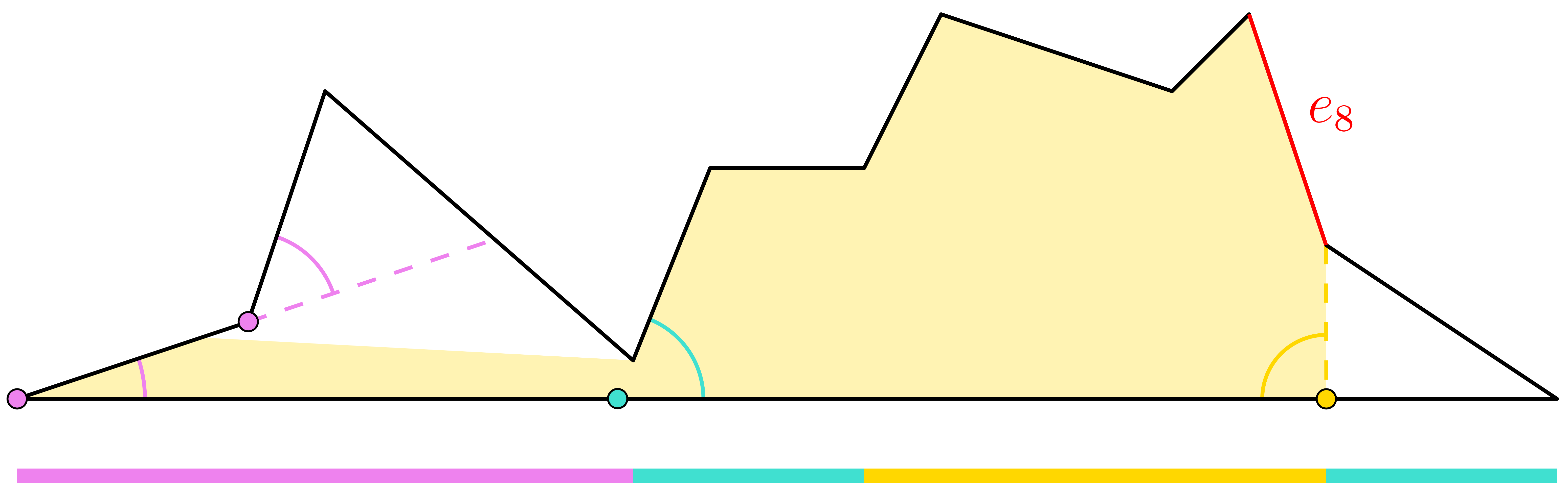

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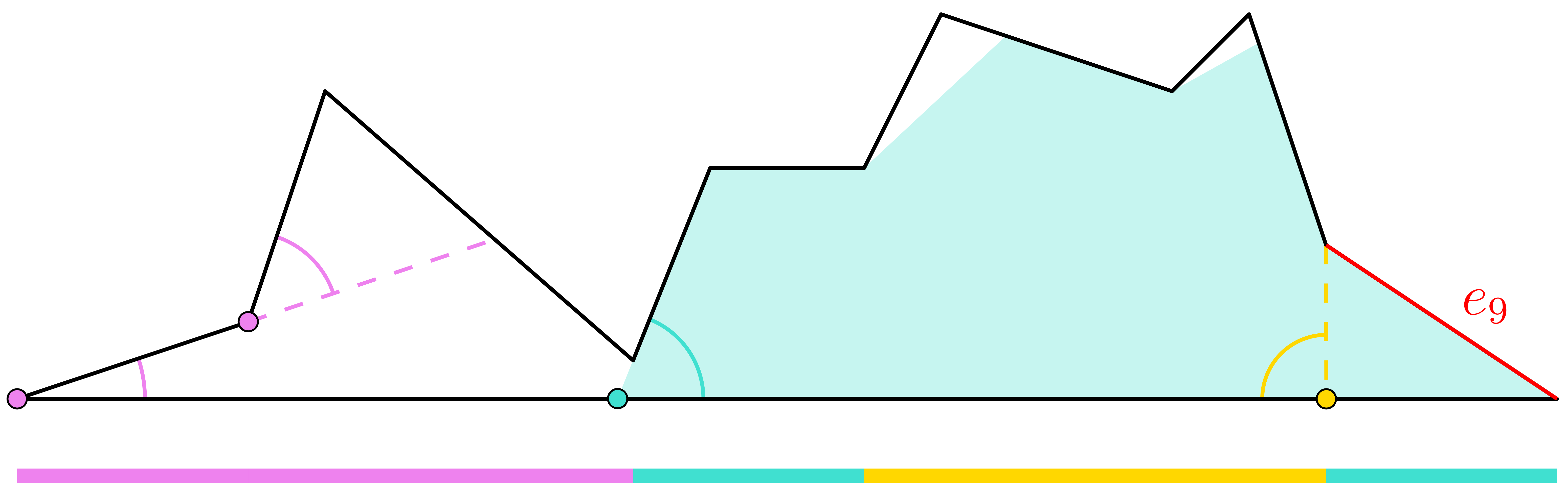

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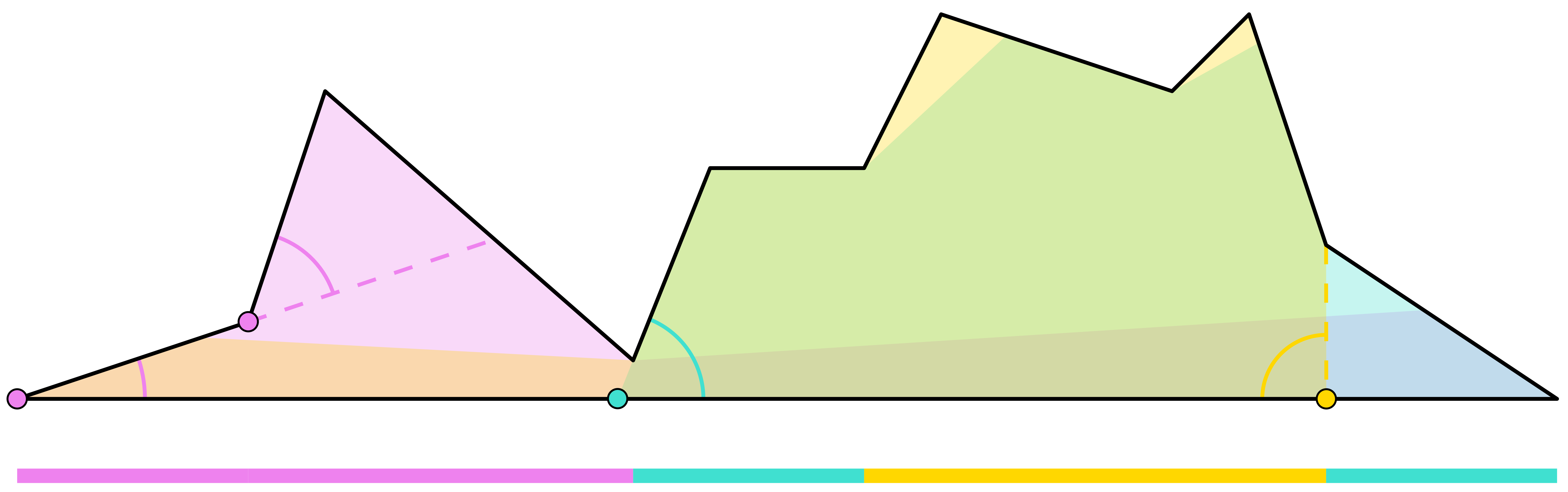

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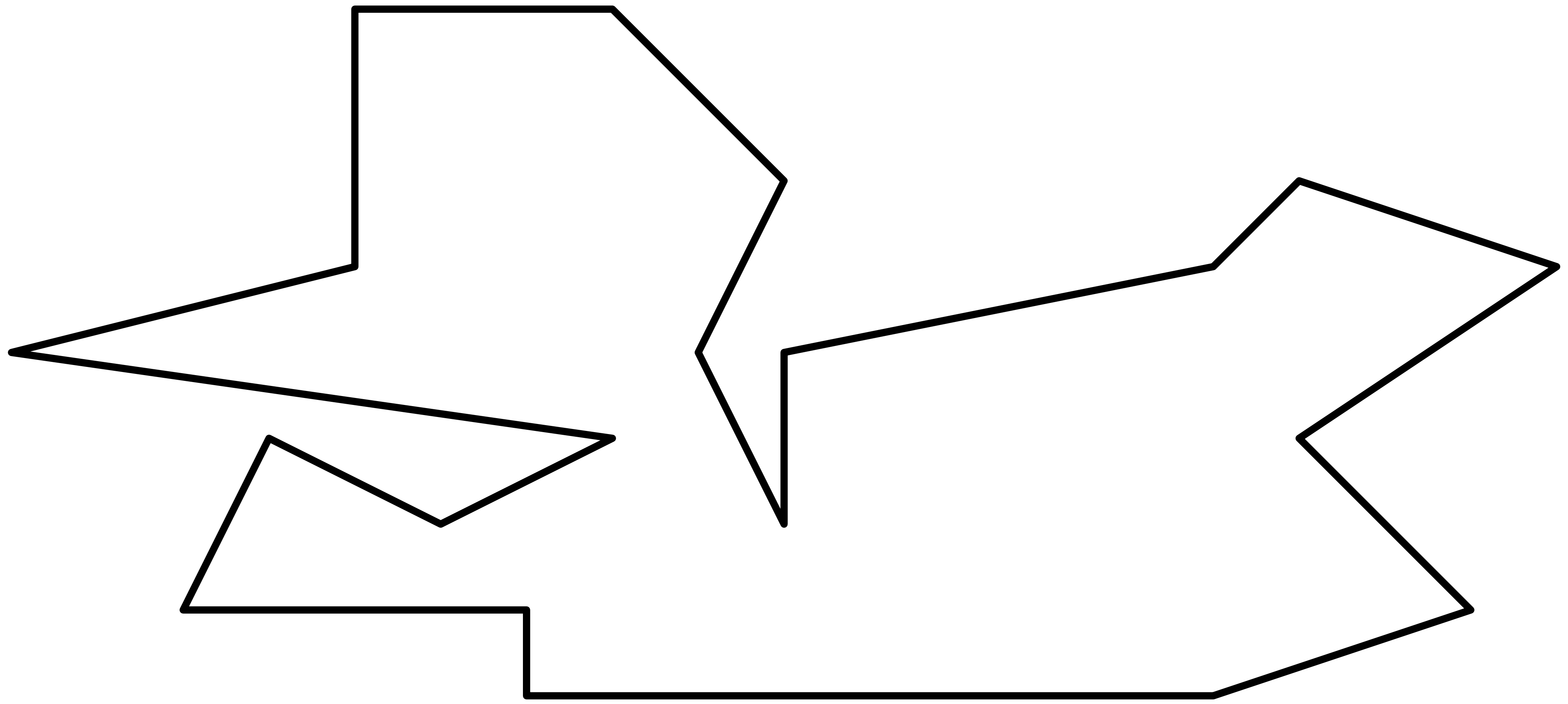

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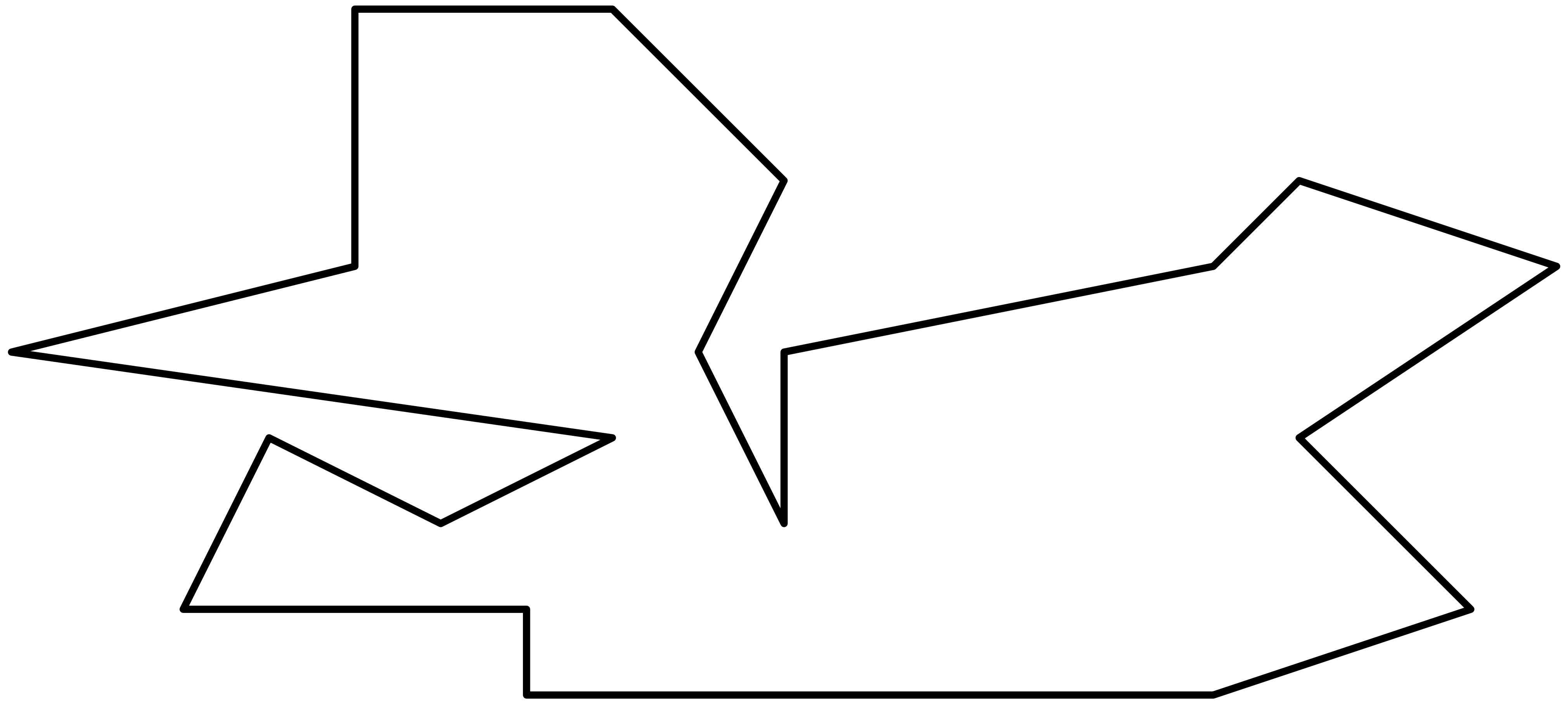
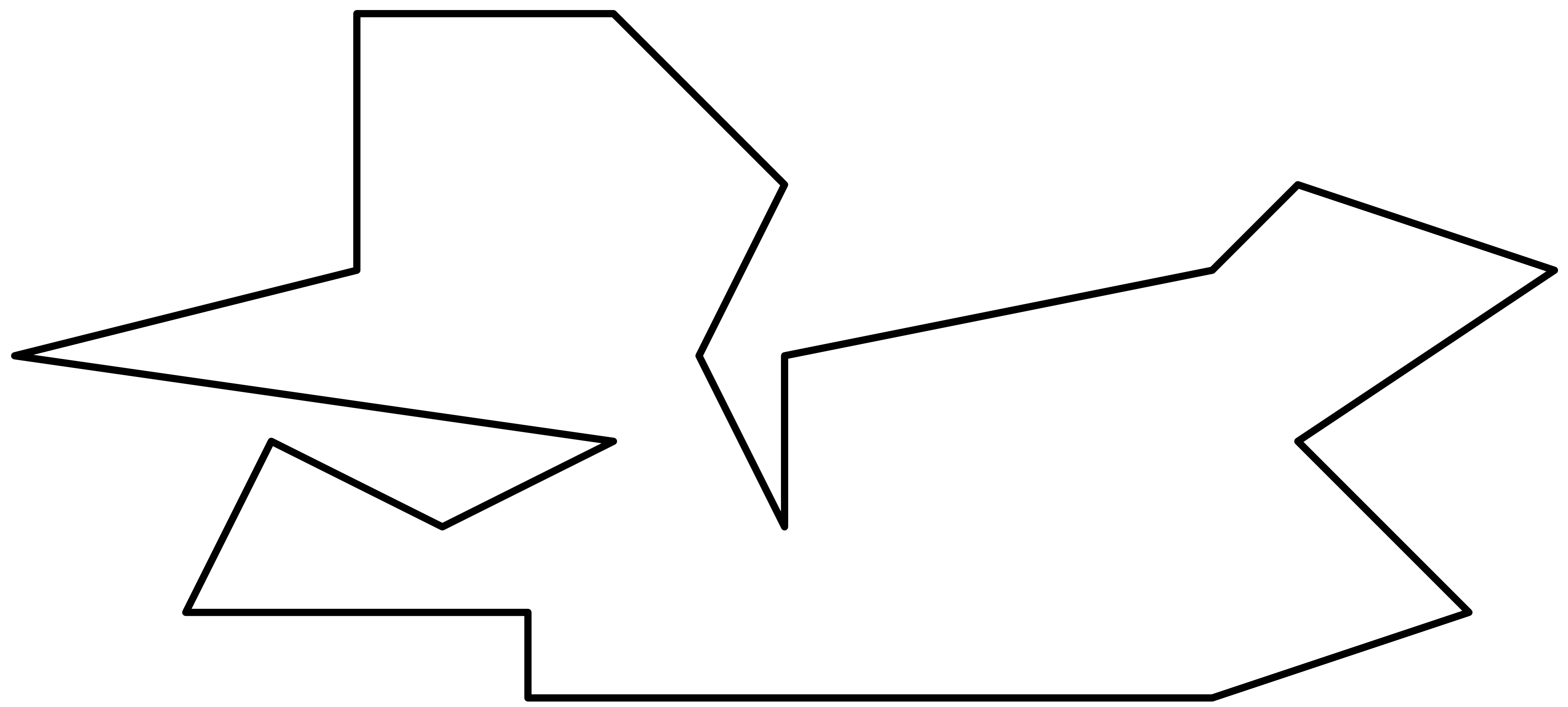
Upper Bound for **Simple Polygons**

Duality

Conclusion

$$(n-2)\frac{\pi}{4}$$





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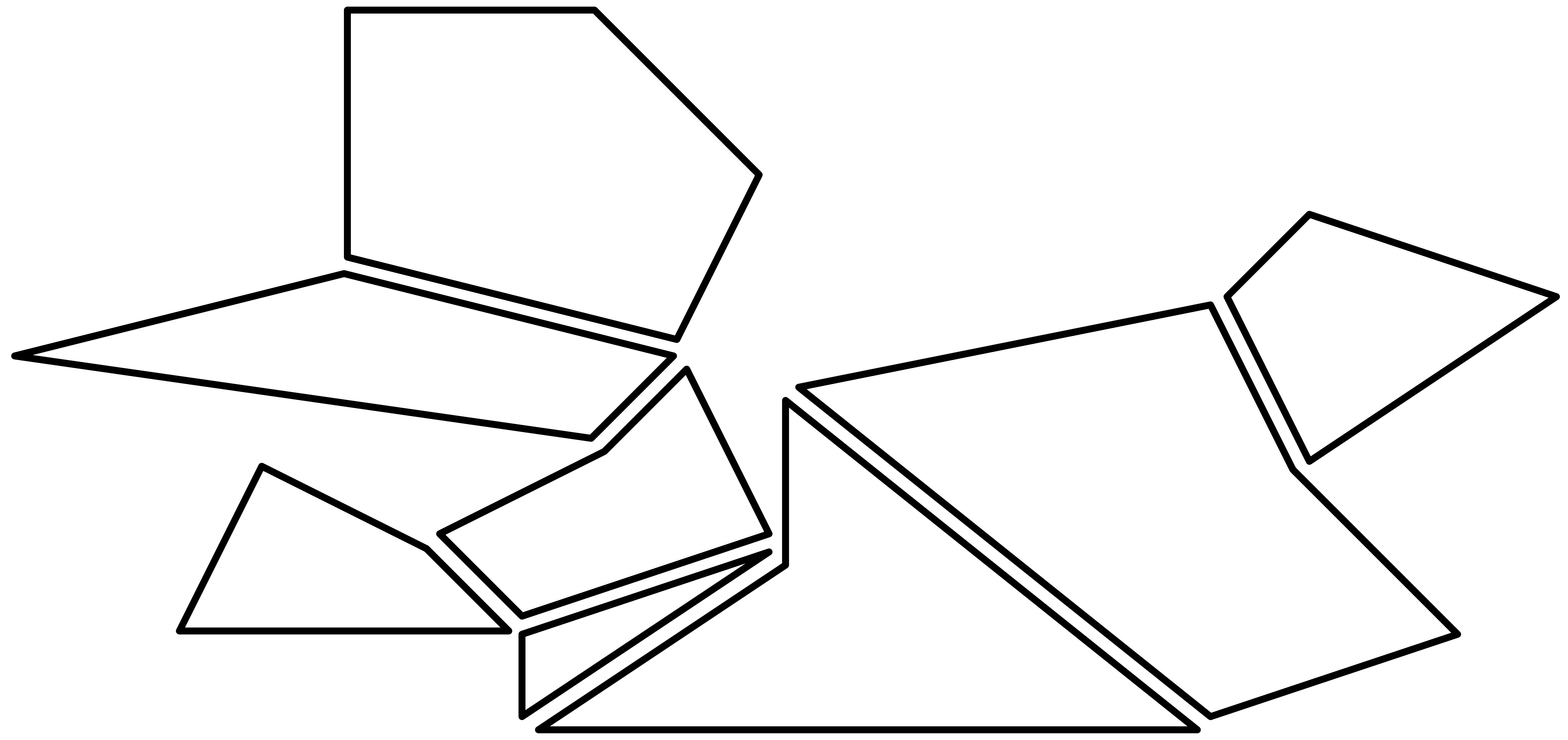

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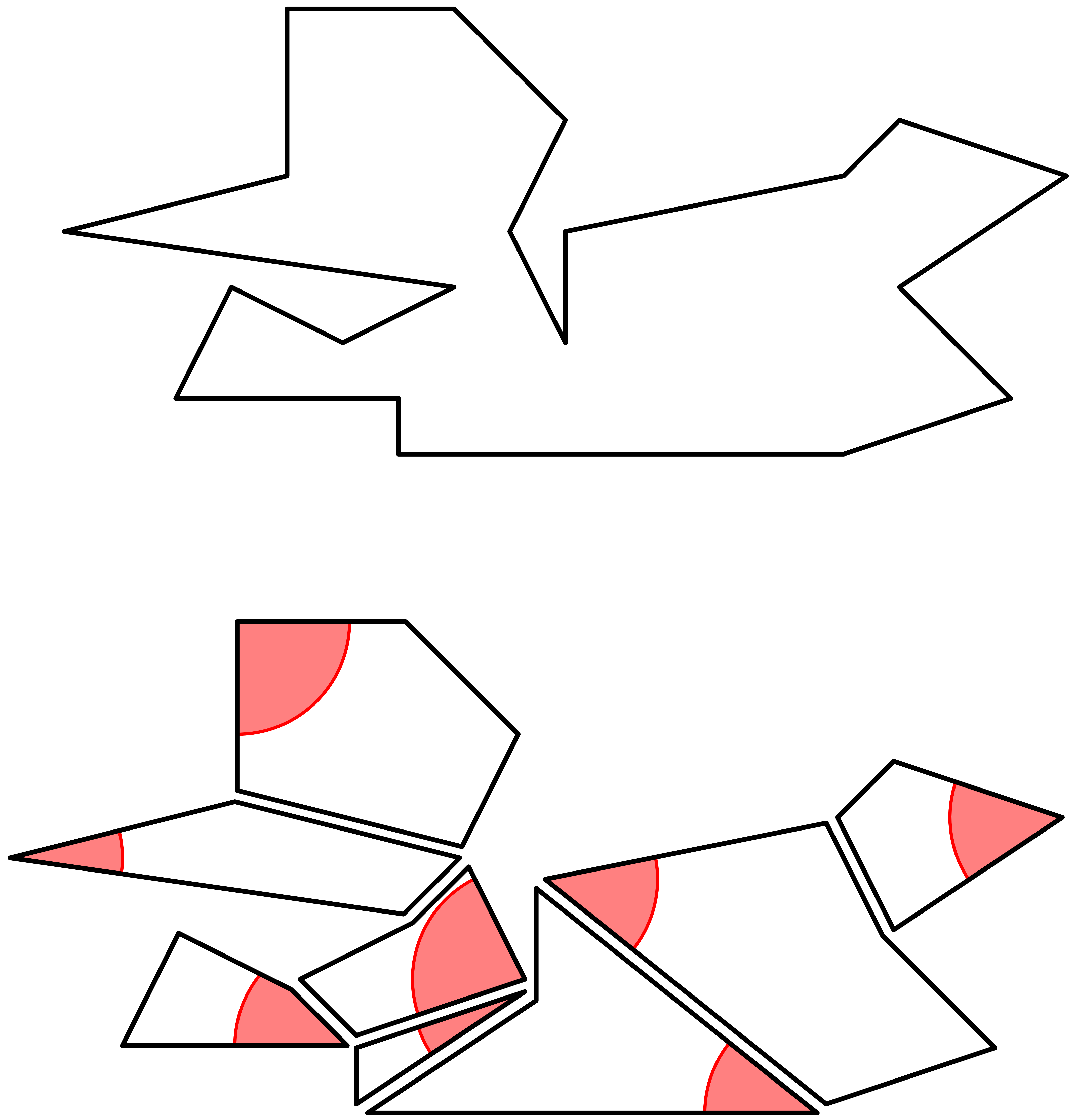

Upper bound for simple polygons

Upper bound for $n \geq k + 2$ of

$$\left(1 + \frac{1}{k}\right)(n-2)\frac{\pi}{6}$$

- Proven for $k = 2$
- Prove for $k = 10$ → $UB \leq 1.1\, LB$

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Duality to independent circle packing

Independent Circle Packing Problem

Instance: A polygon P

Wanted: A set of independent circles in P

Maximize: The minimum required angle to cover all circles

Duality to independent circle packing

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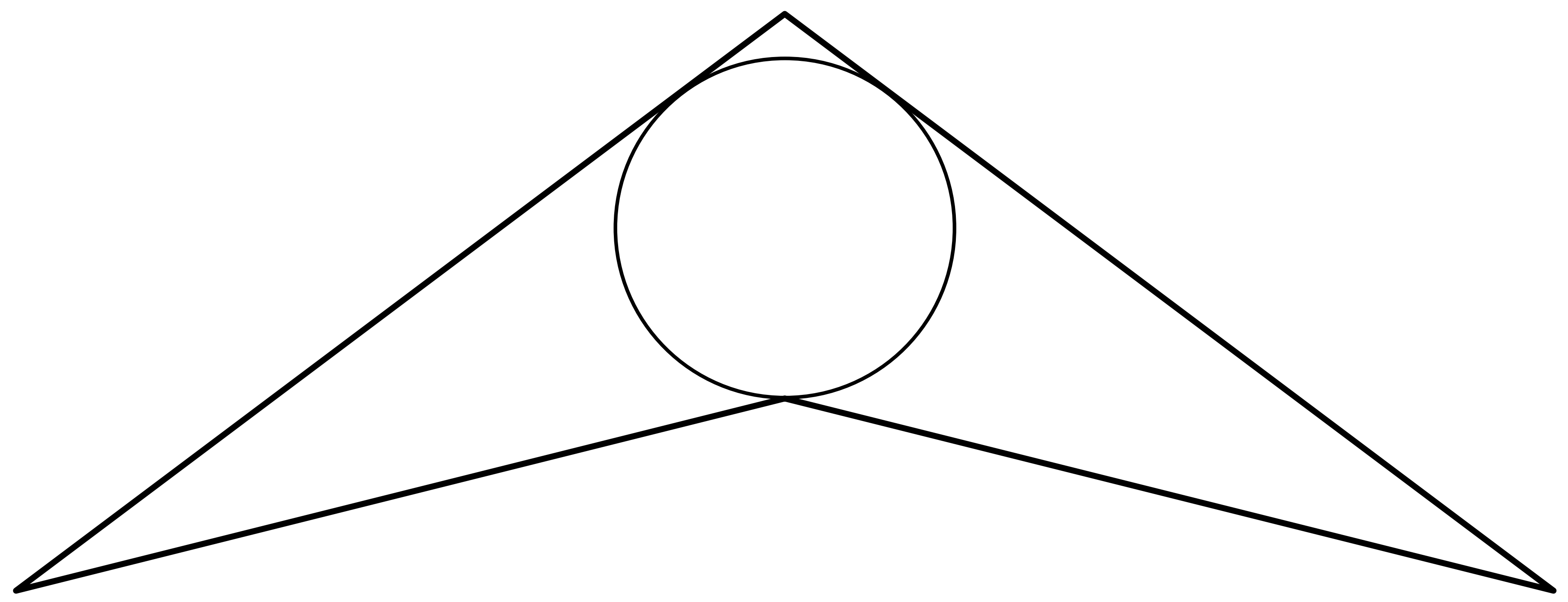

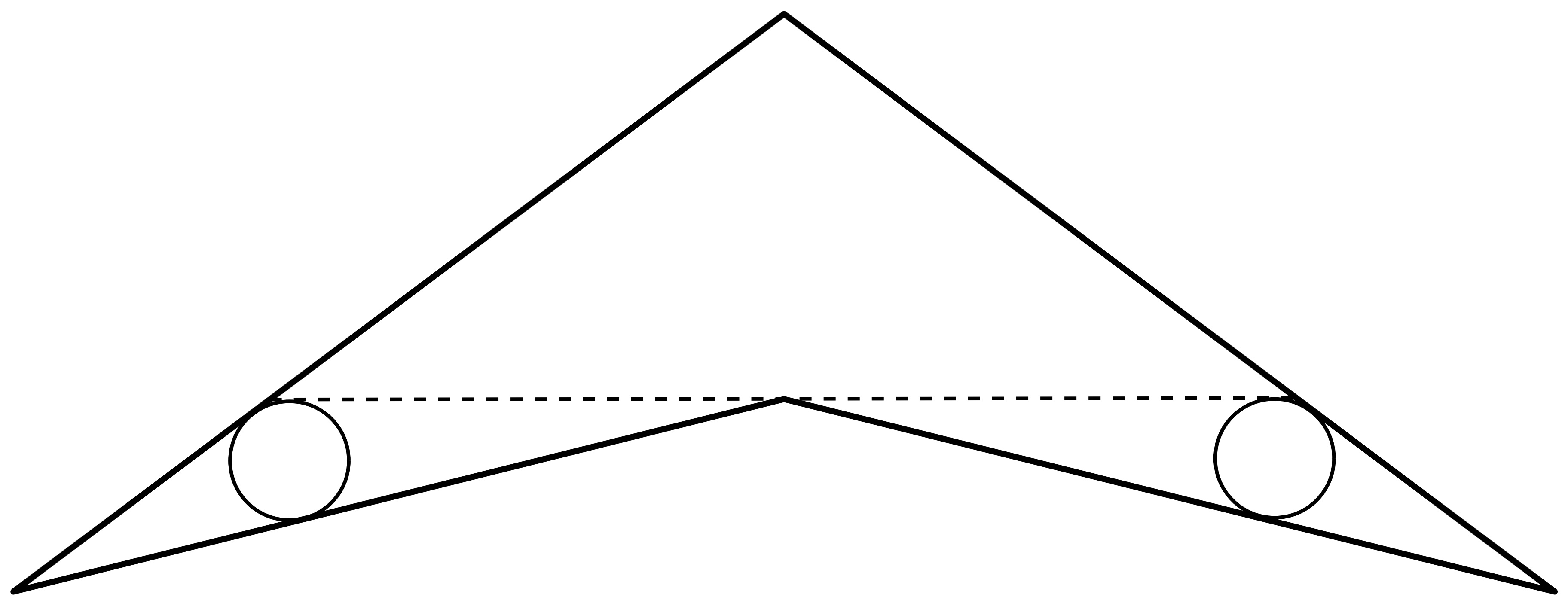

Duality to independent circle packing

Independent Circle Packing Problem

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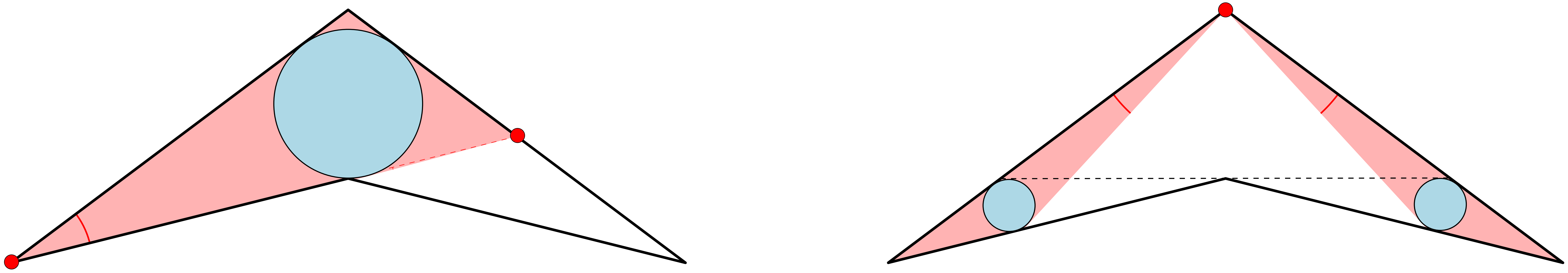
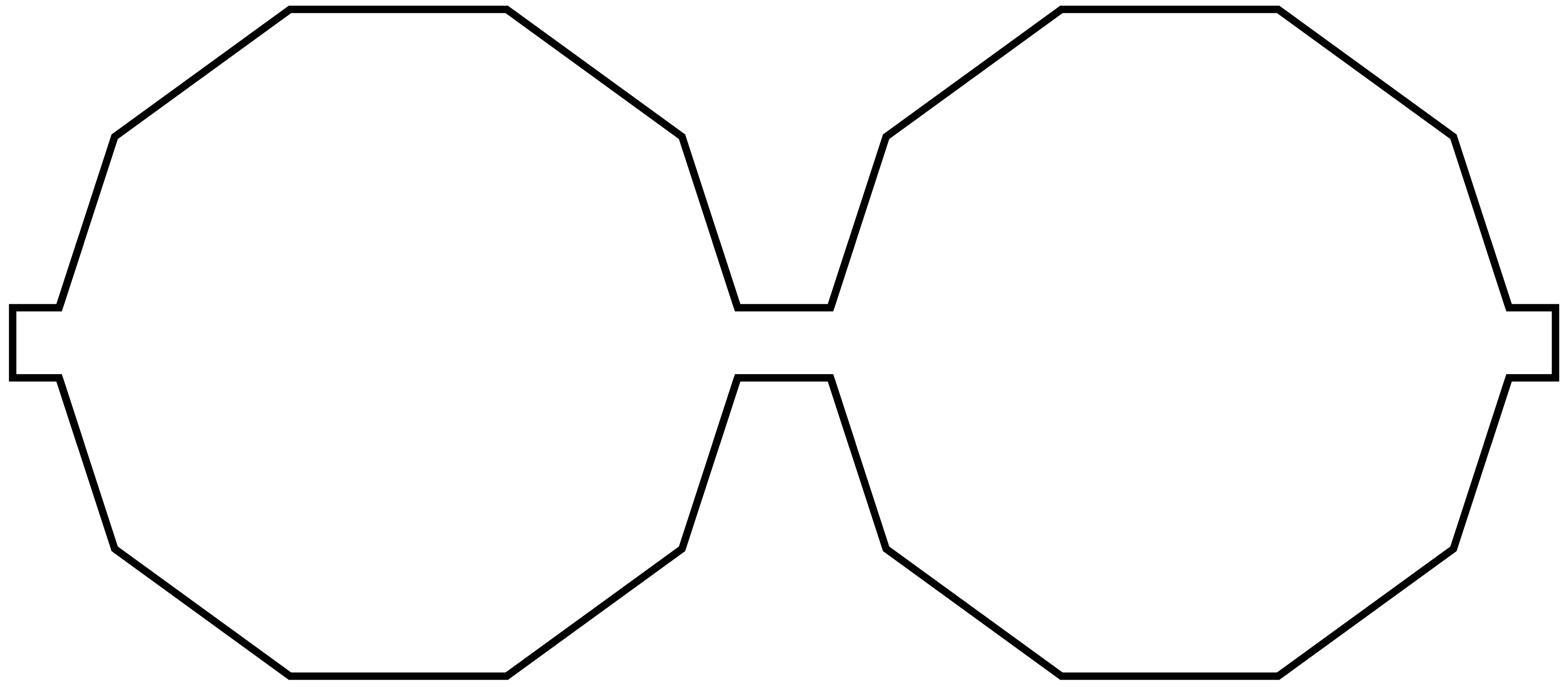
Wanted: A set of independent circles in *P*

Maximize: The minimum required angle to cover all circles



Duality Gap



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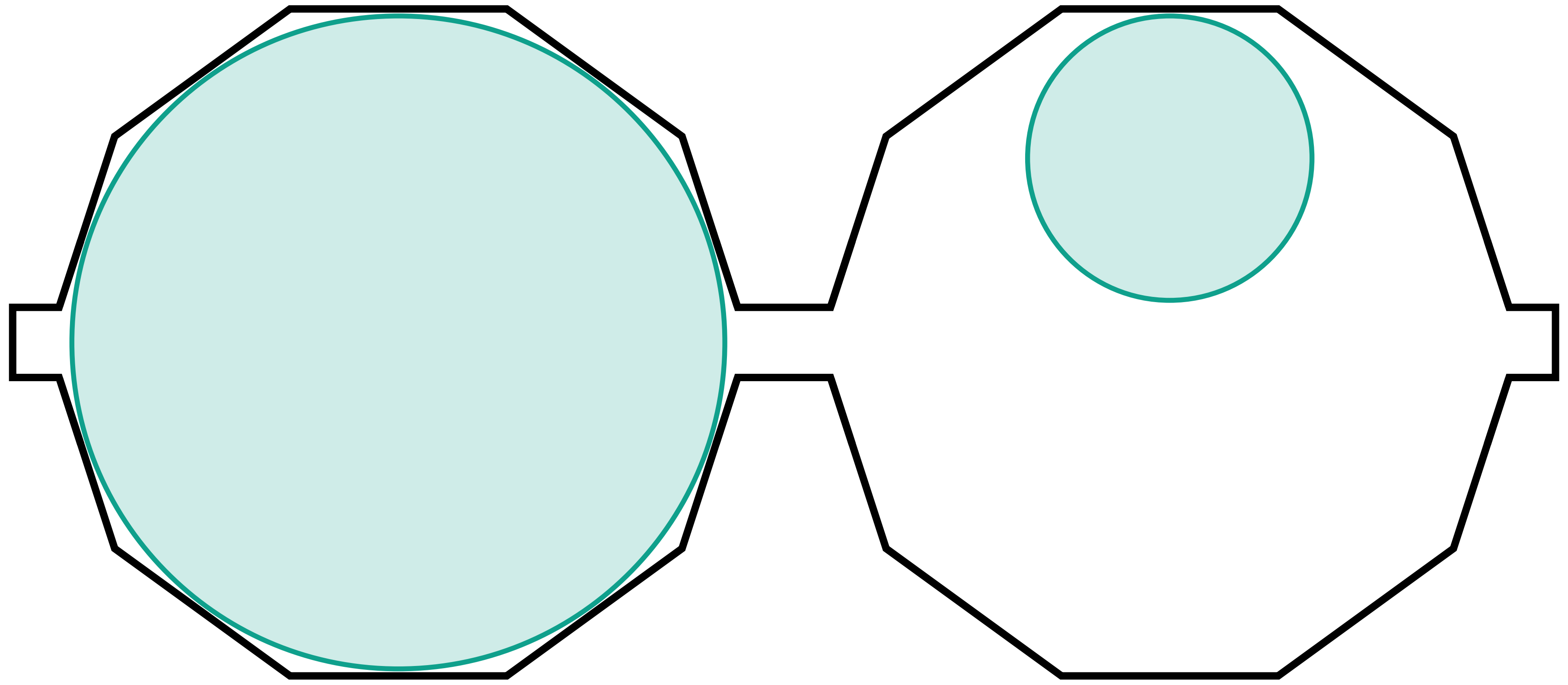

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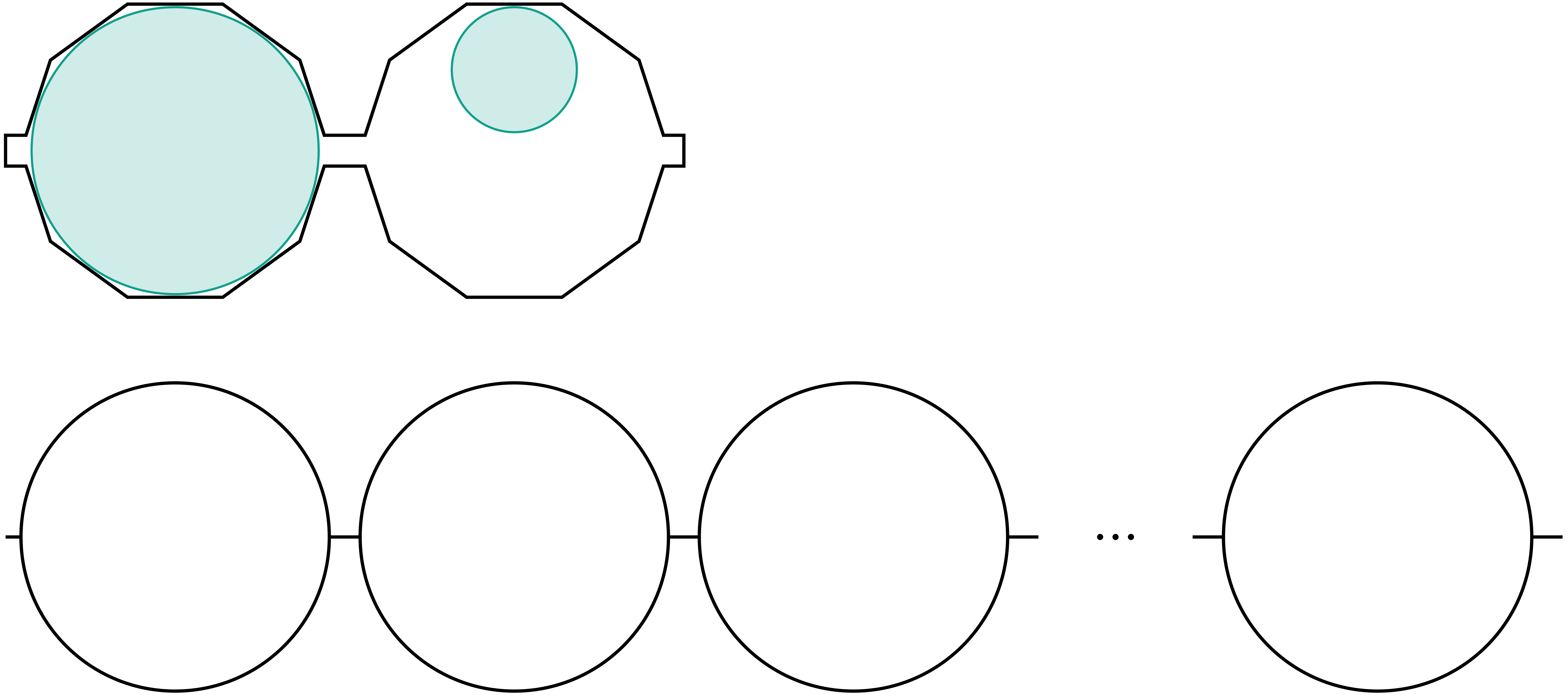

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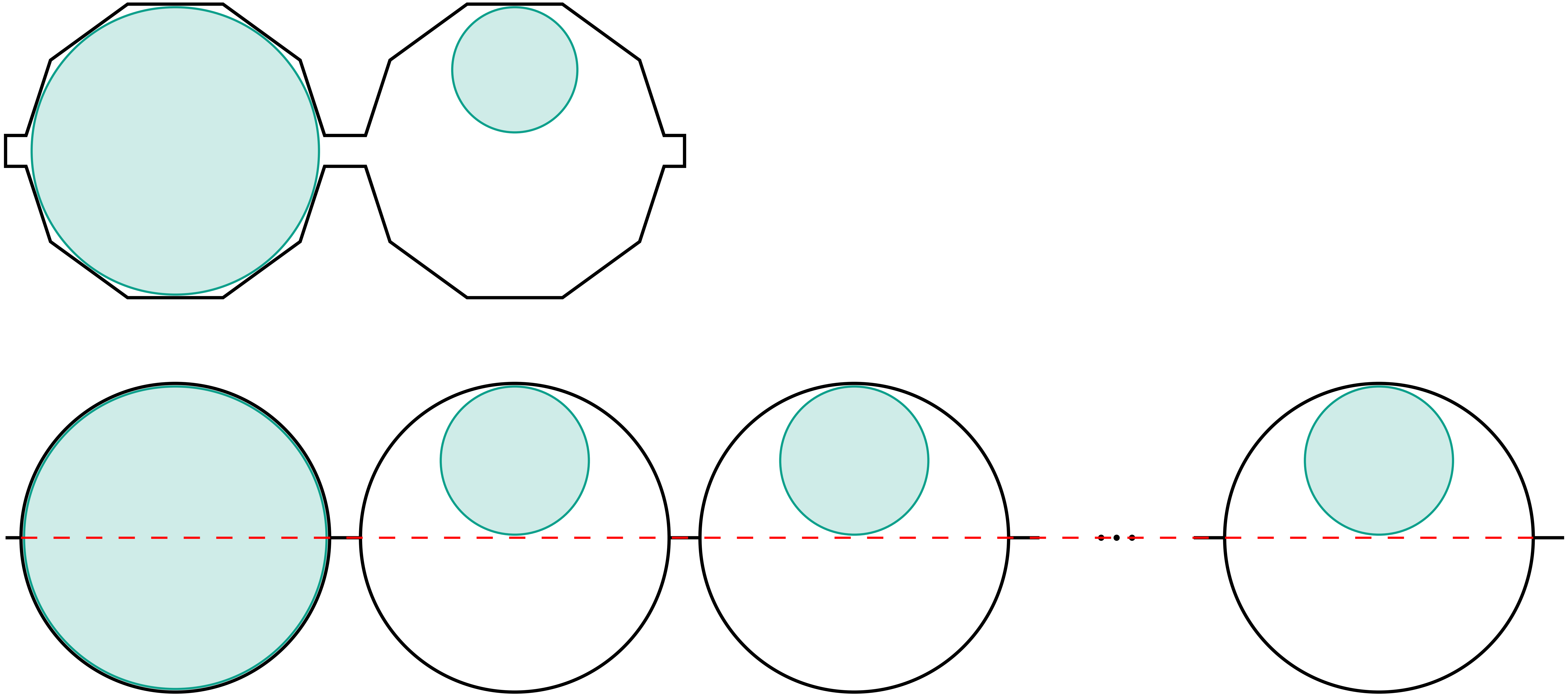

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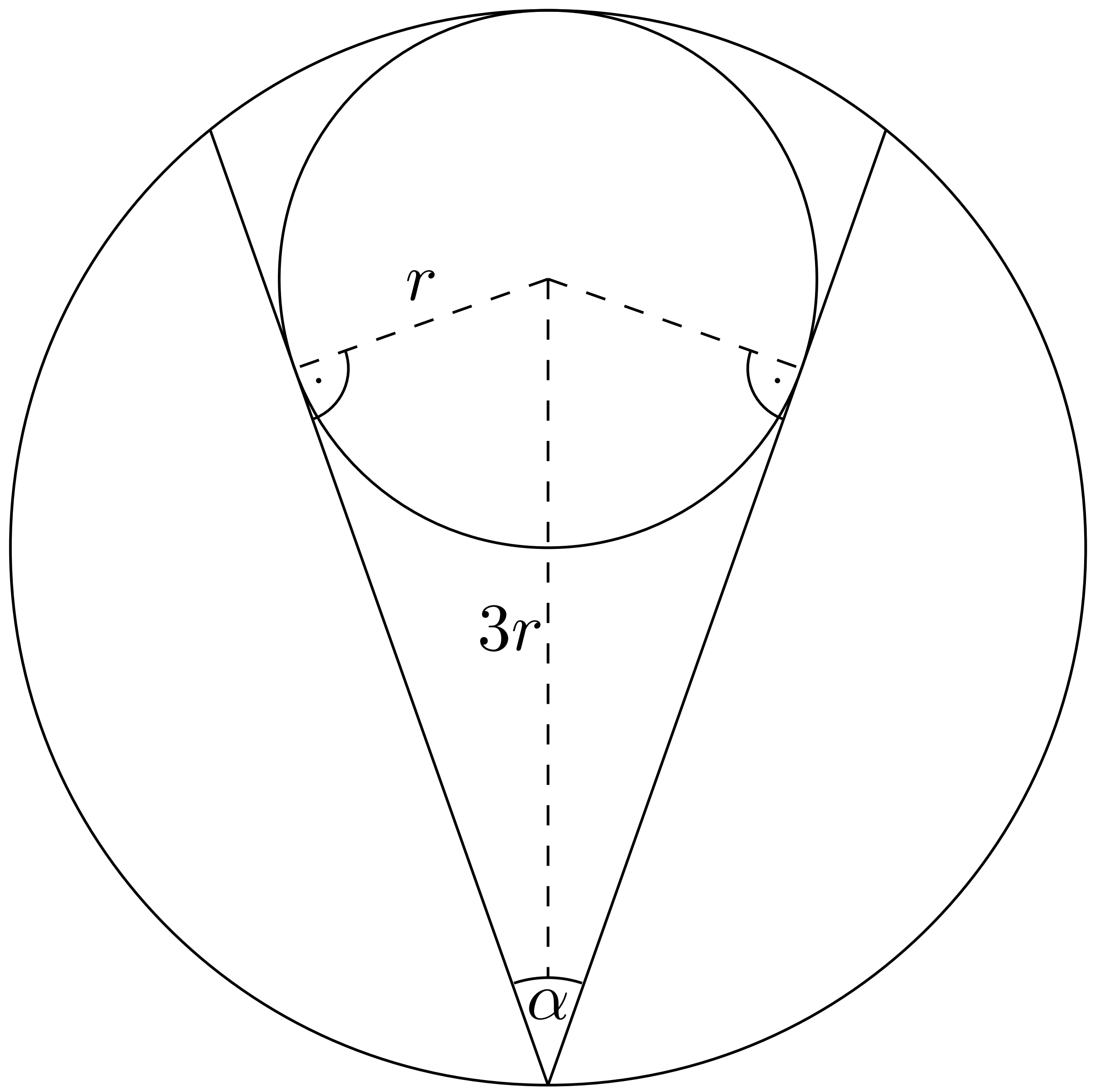
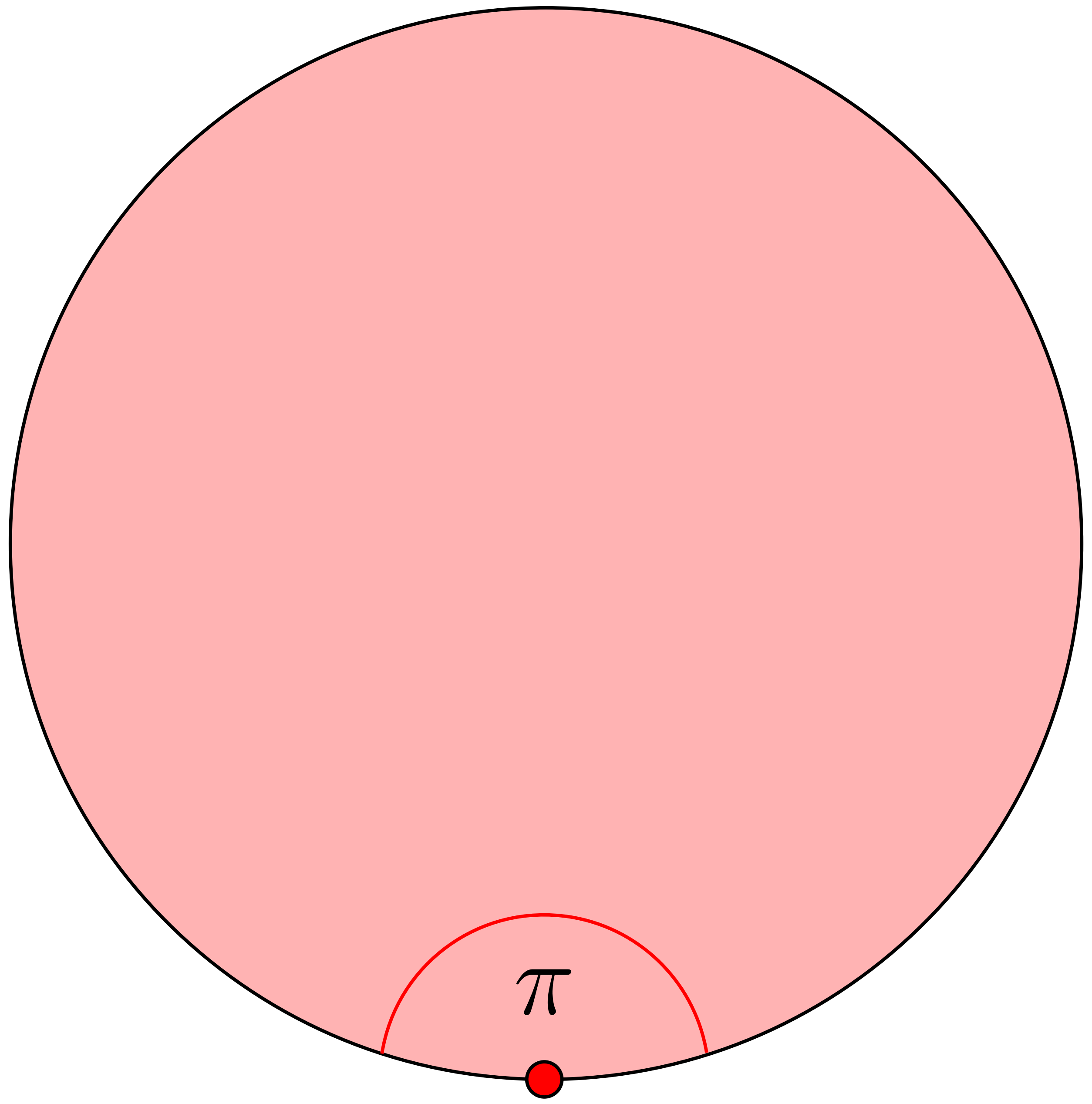


$$\text{ICP} = \alpha = 2\ \arcsin\frac{1}{3}$$



$$\text{AAGP} = \pi$$

$$\frac{\text{AAGP}}{\text{ICP}} = \frac{\pi}{2\ \arcsin\frac{1}{3}} \approx 4.622$$

Conclusion

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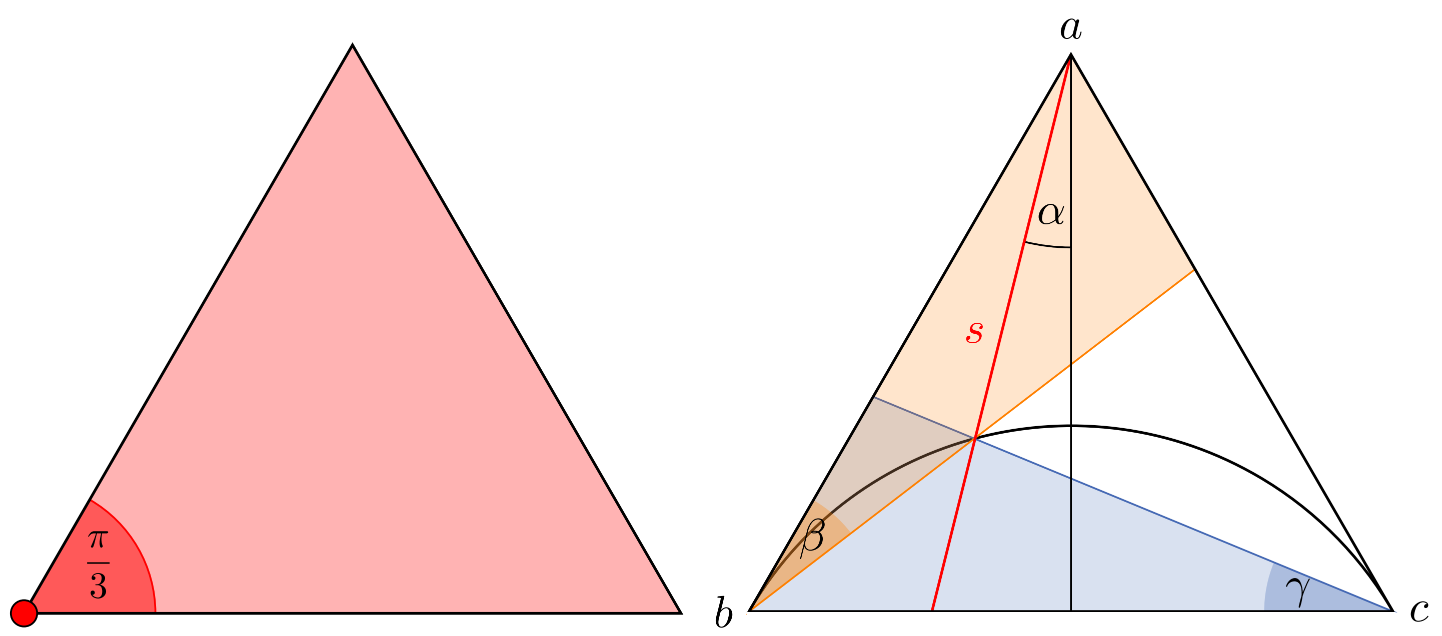
An optimal covering of an equilateral triangle has an angle of

$$\frac{\pi}{3}$$



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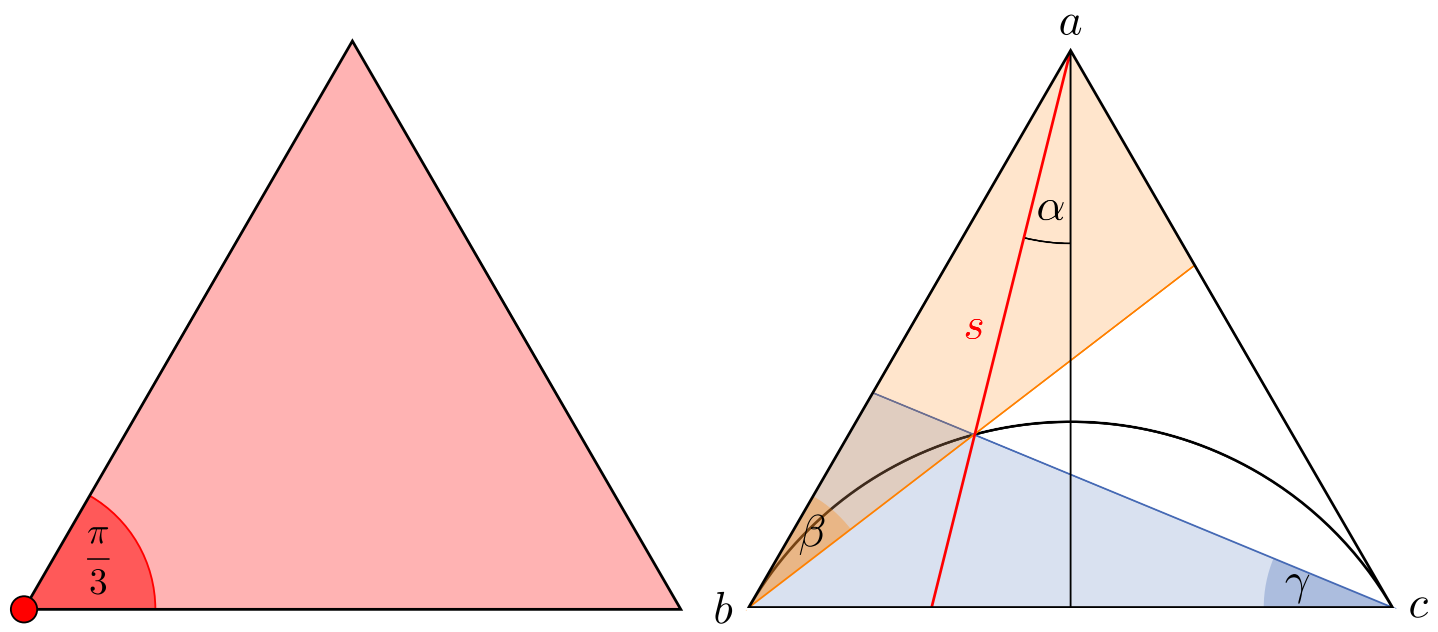

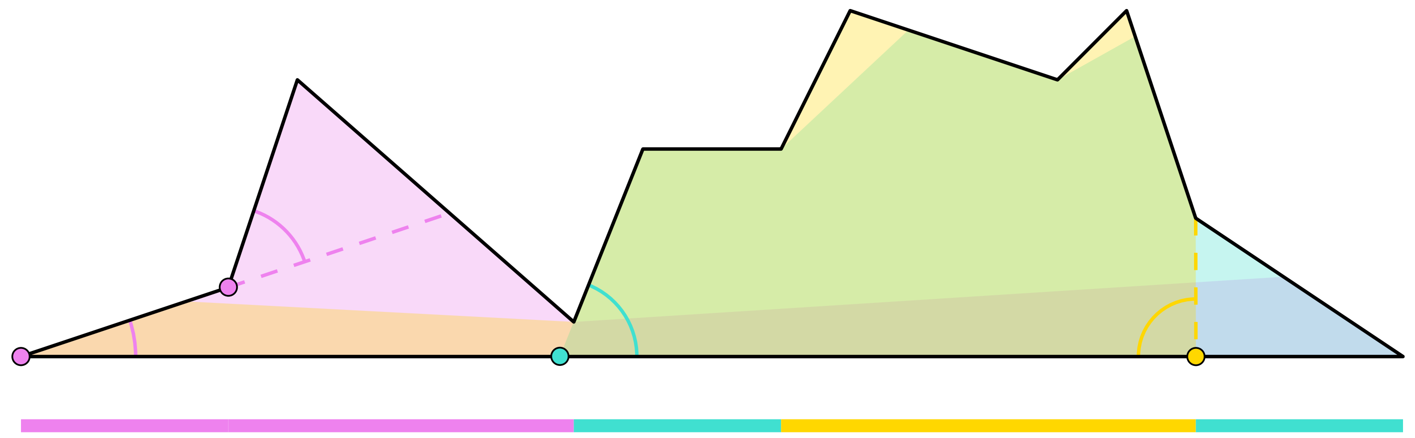

There is an upper bound of

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sufficient to cover any given histogram polygon.

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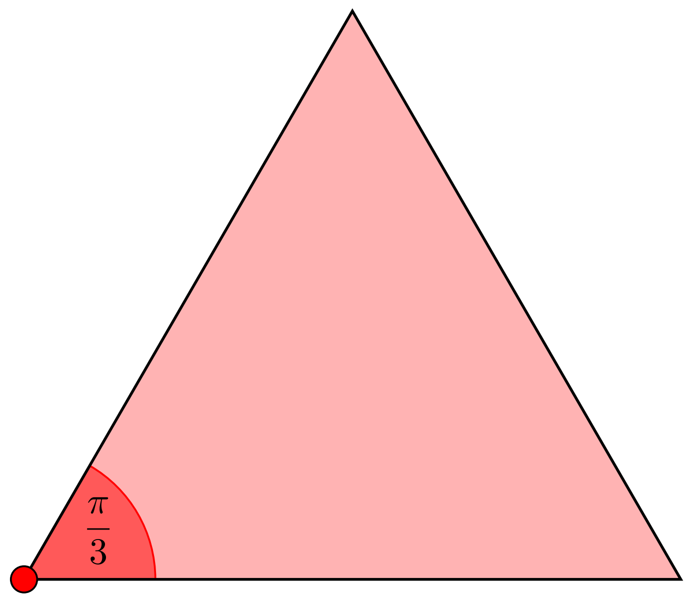

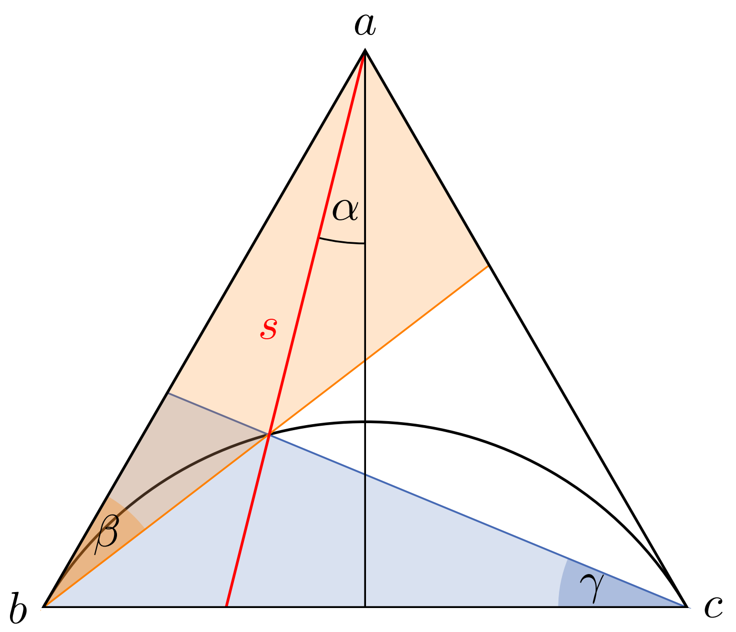

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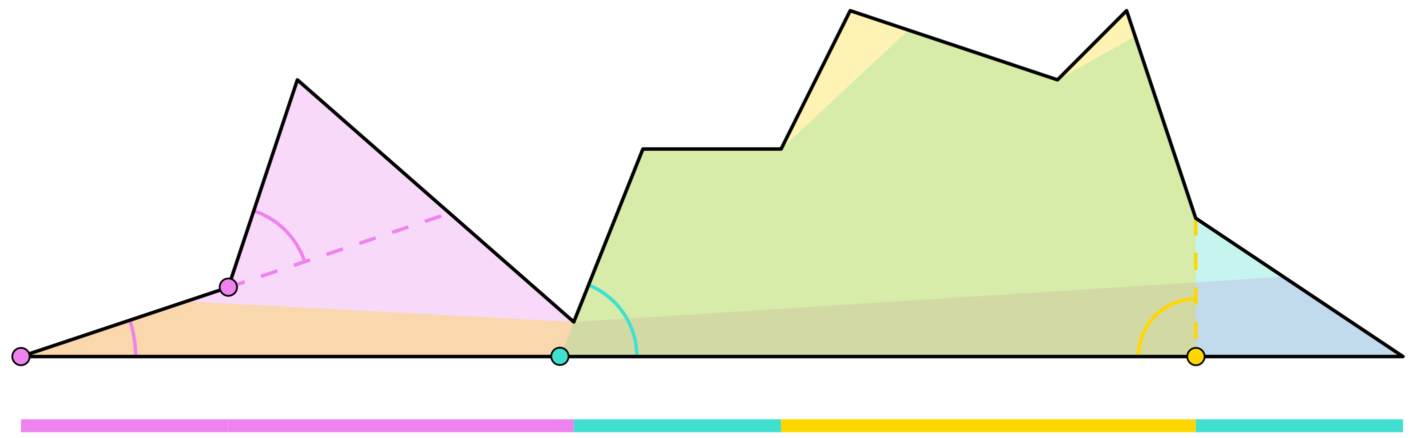

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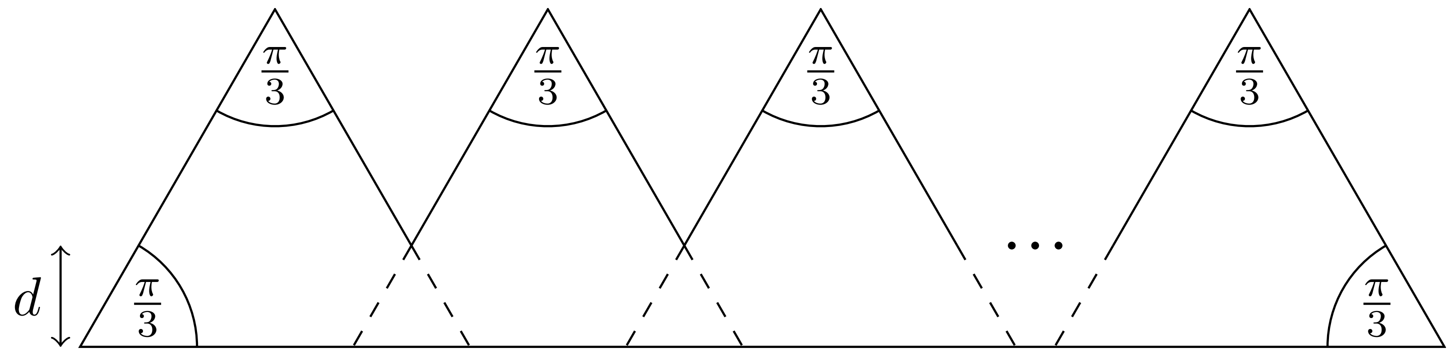

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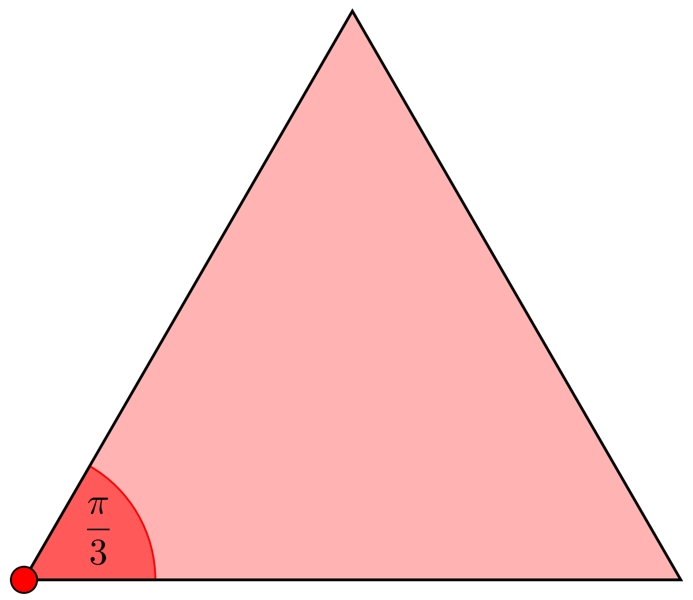

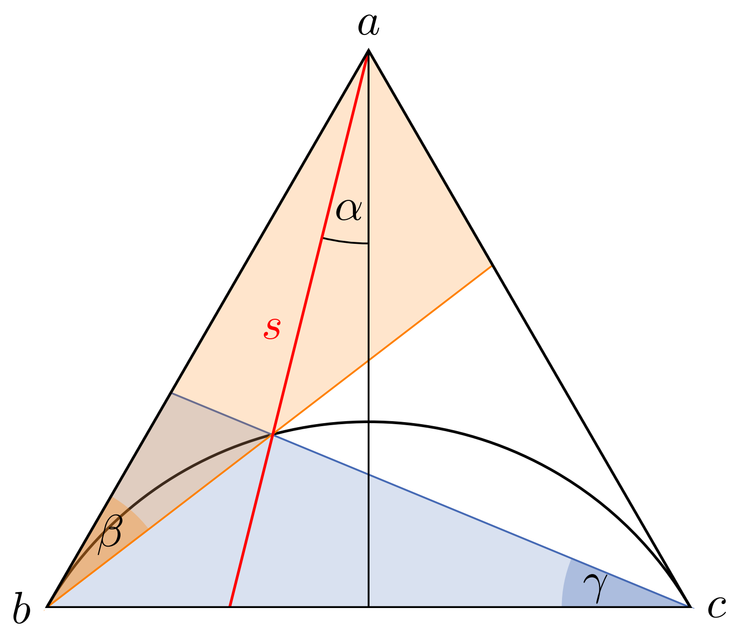

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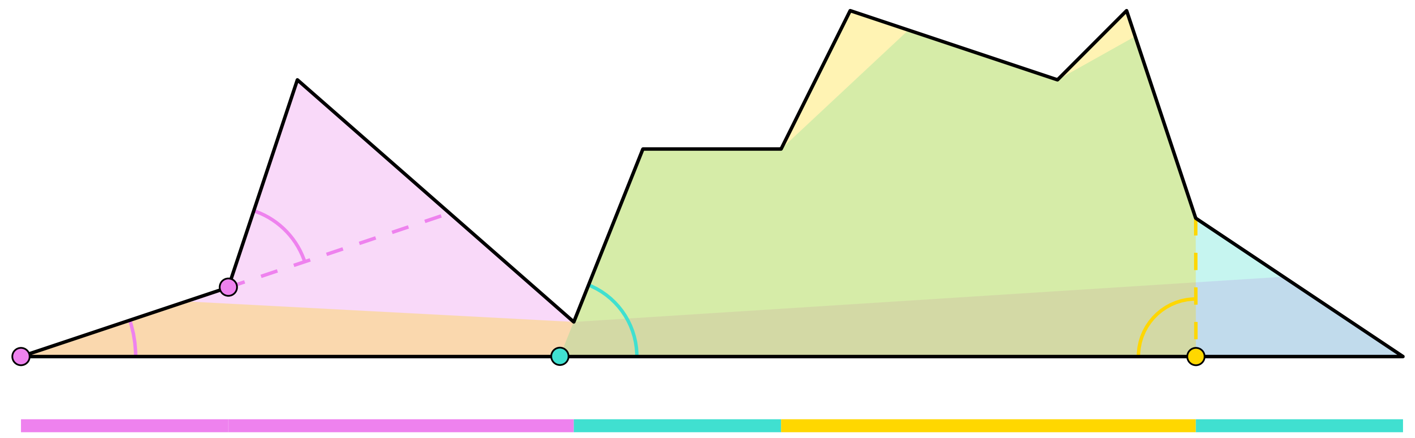

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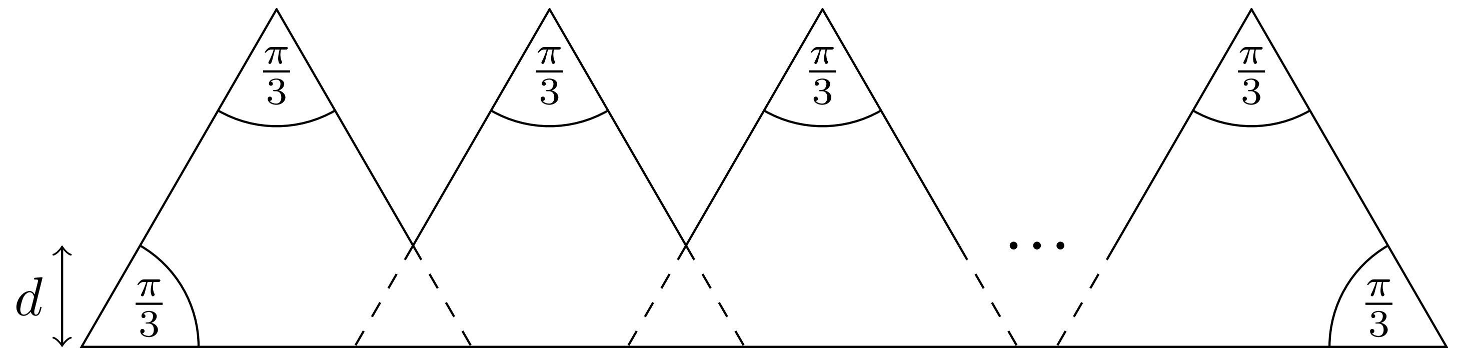

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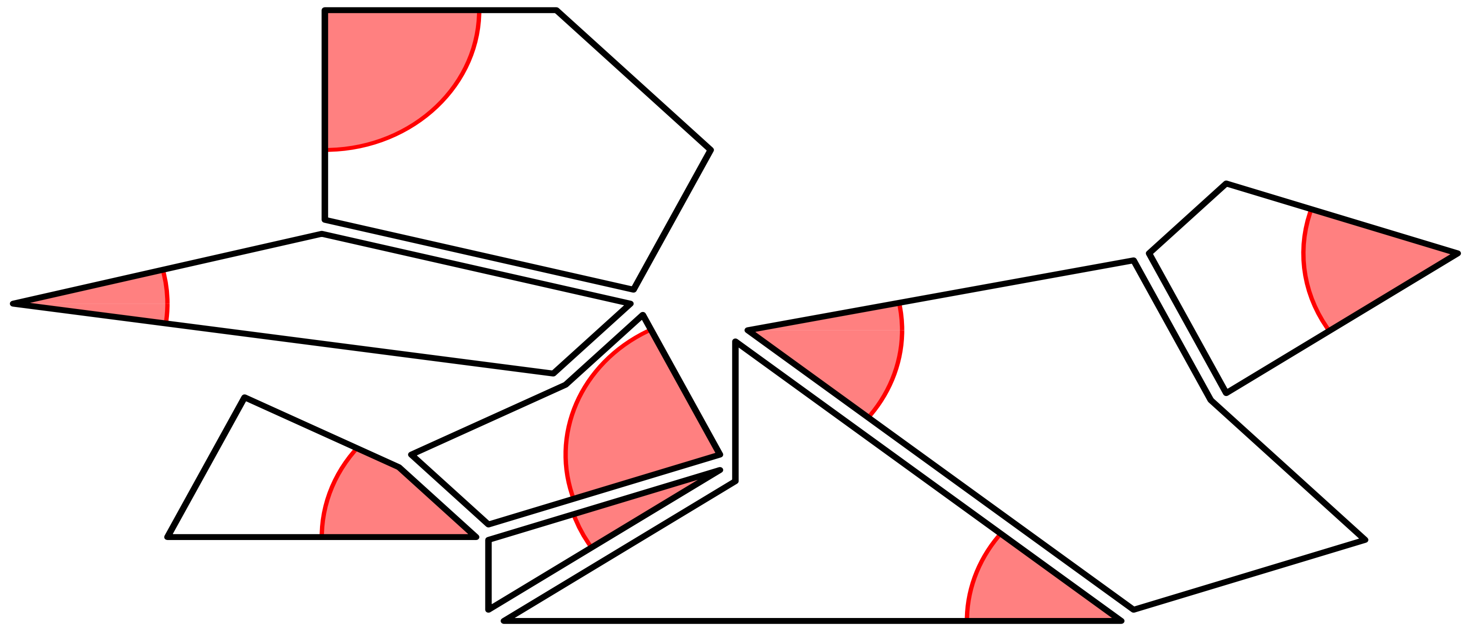

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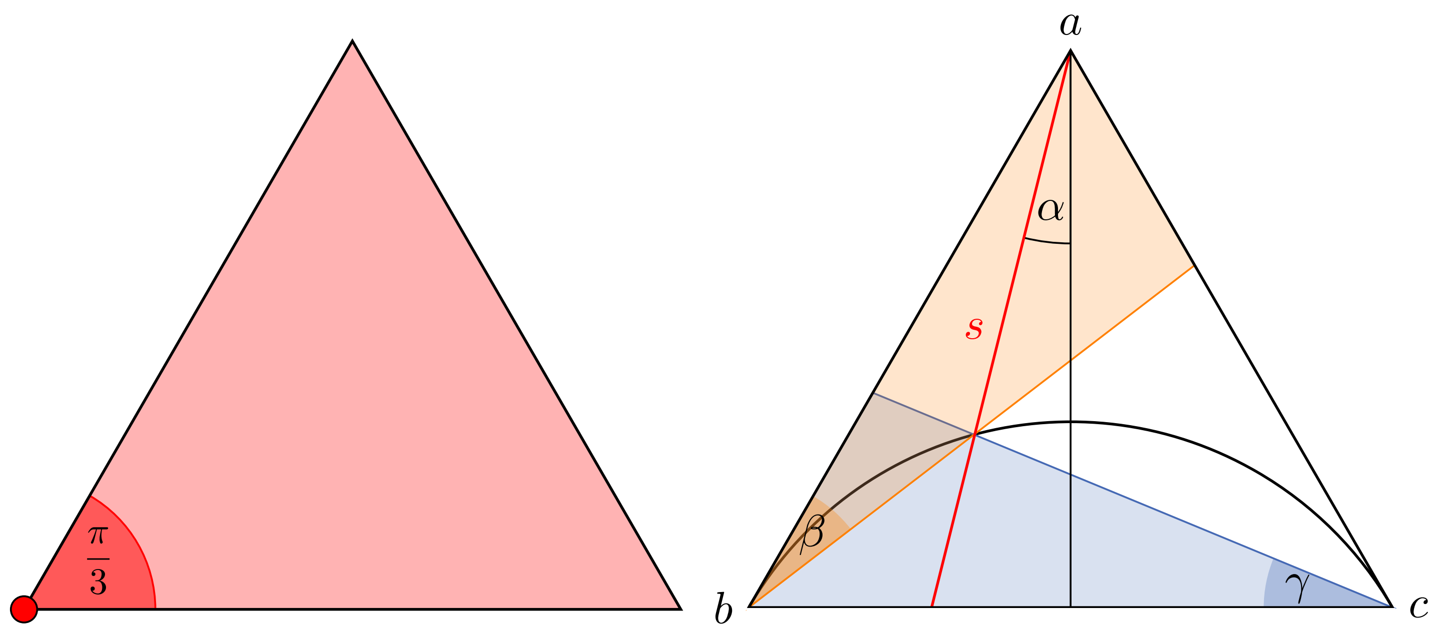

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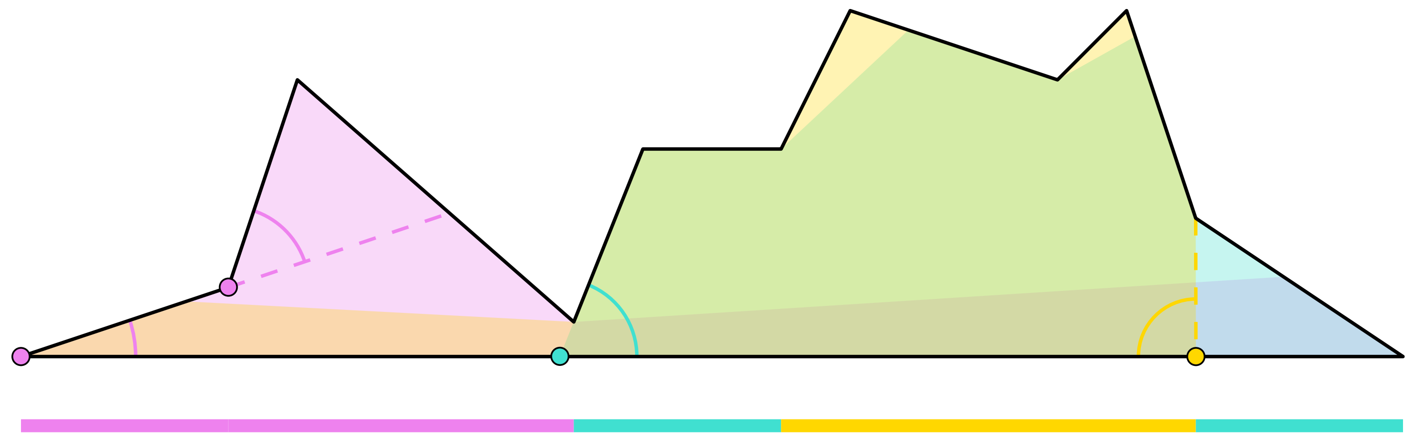

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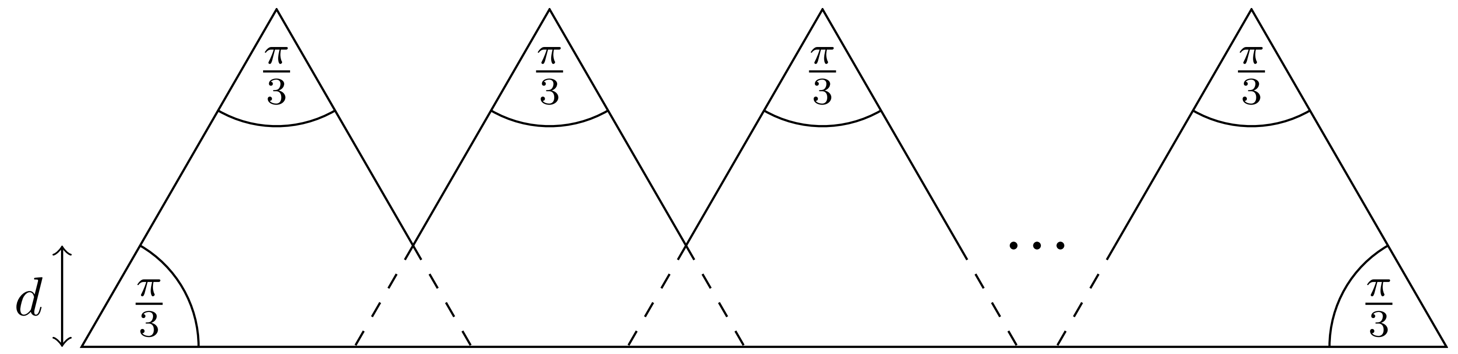

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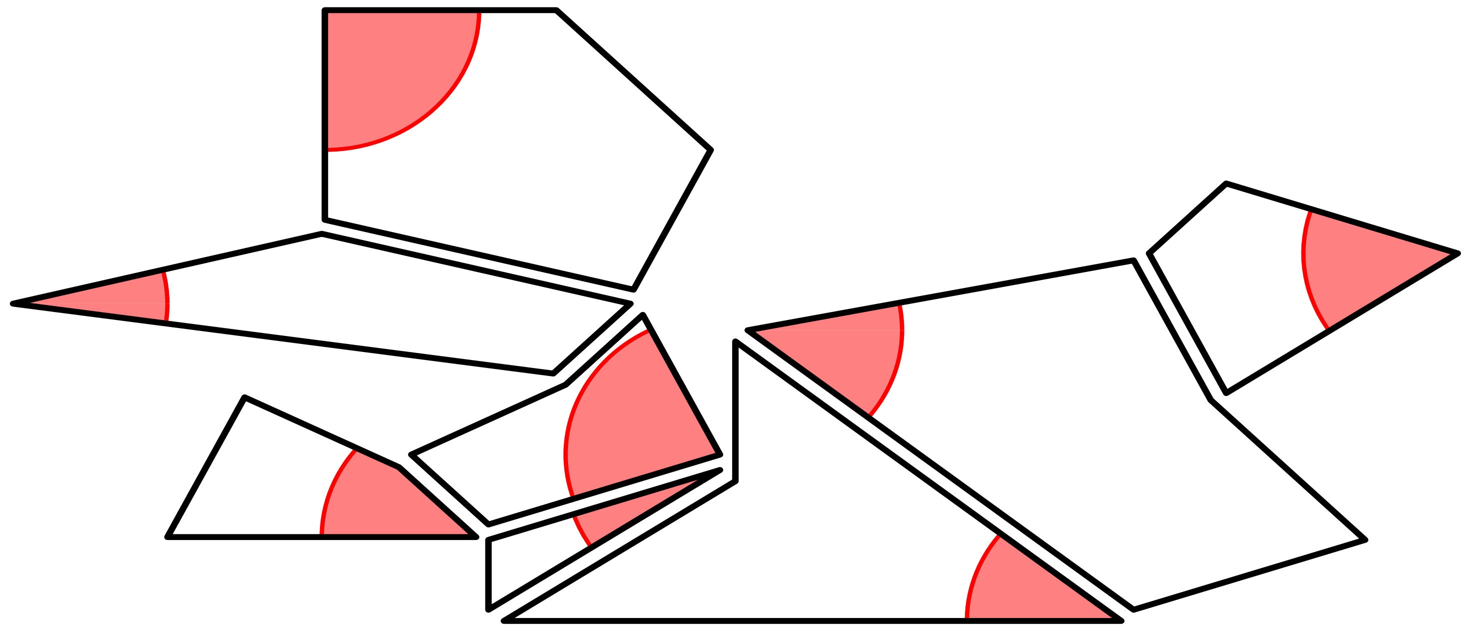

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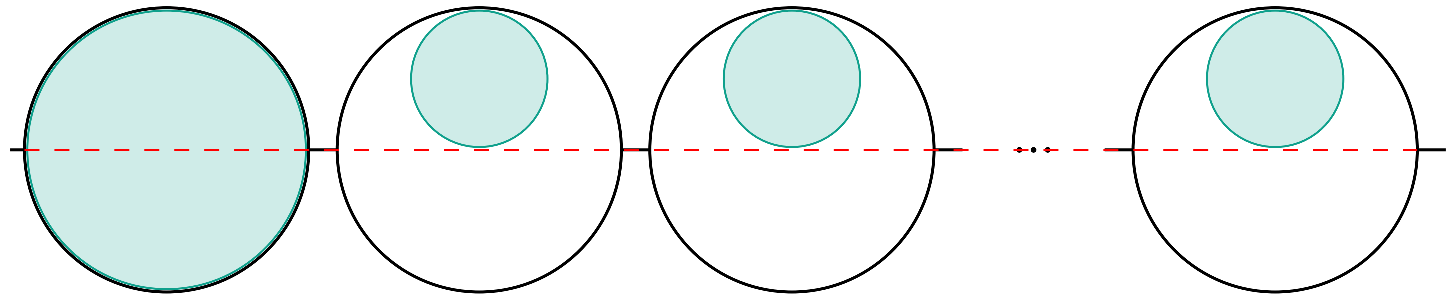
We determined a duality gap of

$$\approx 4.662$$

between ICP and AAGP.



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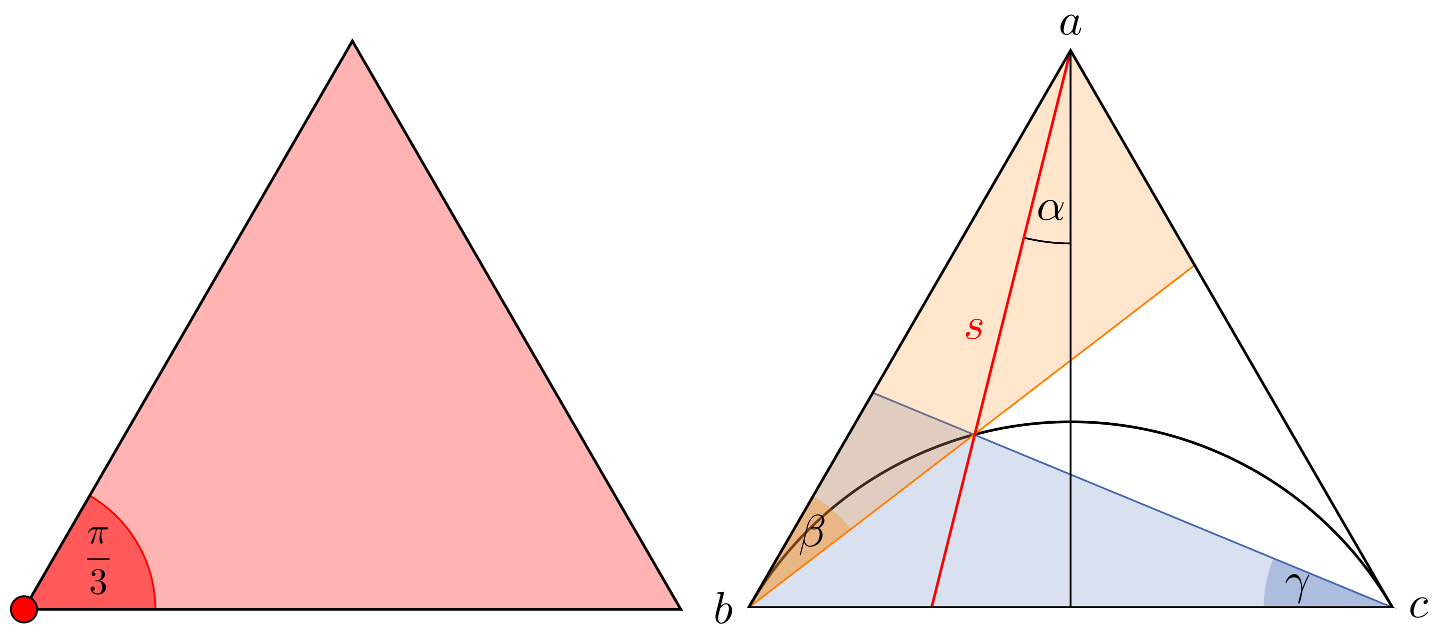

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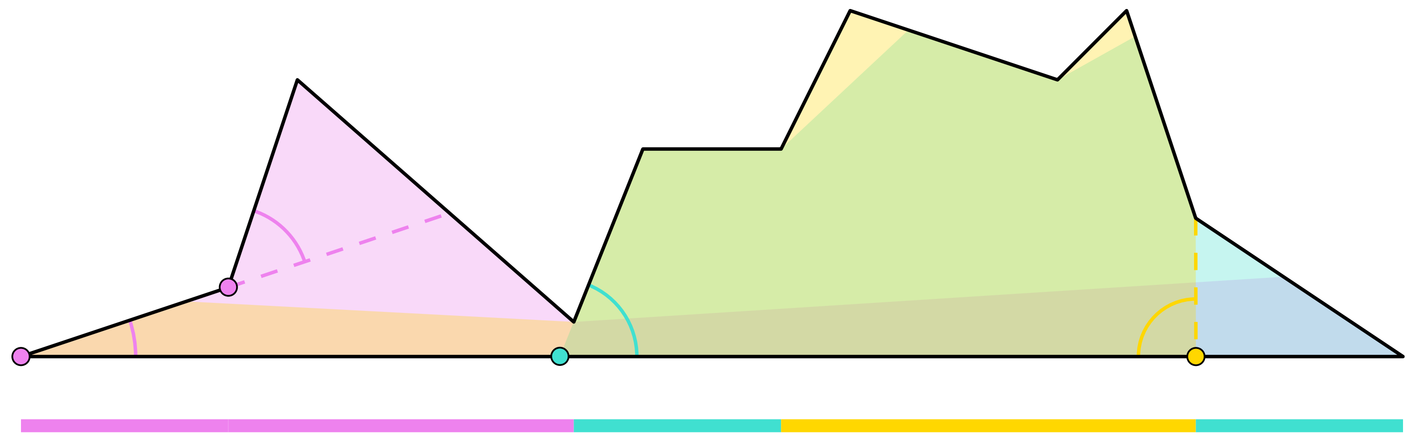

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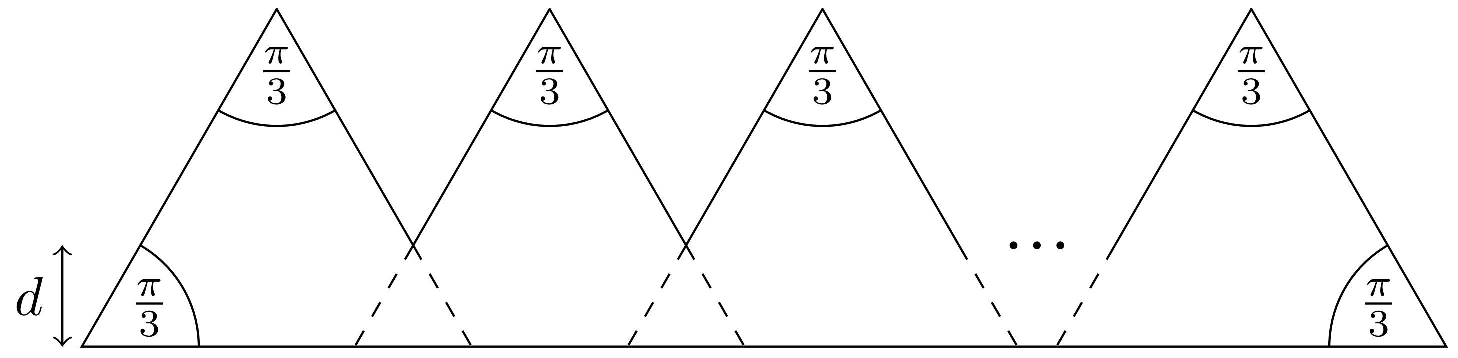

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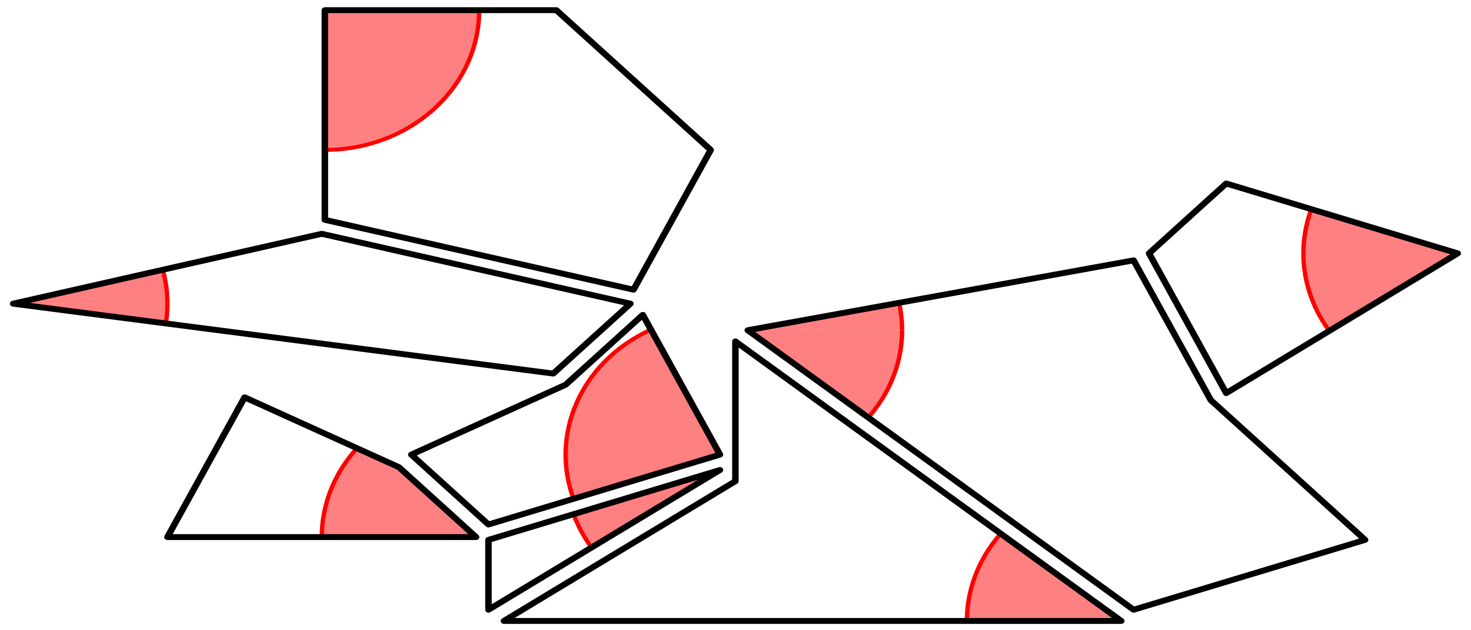

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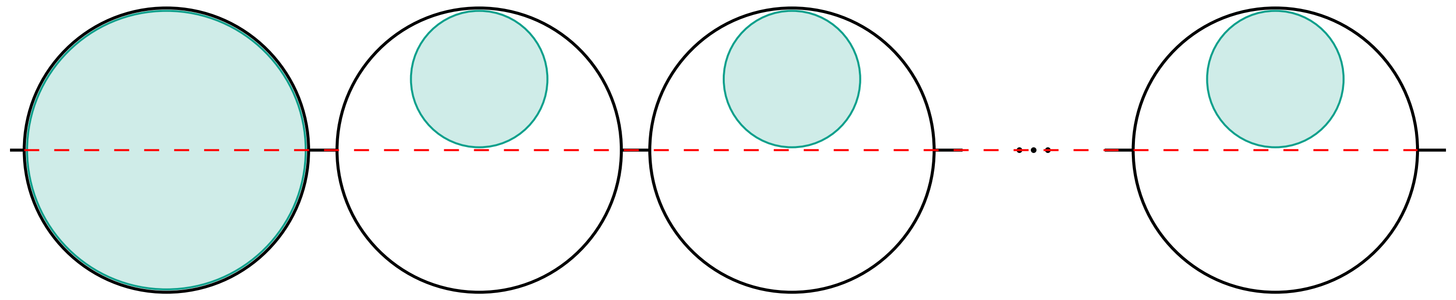
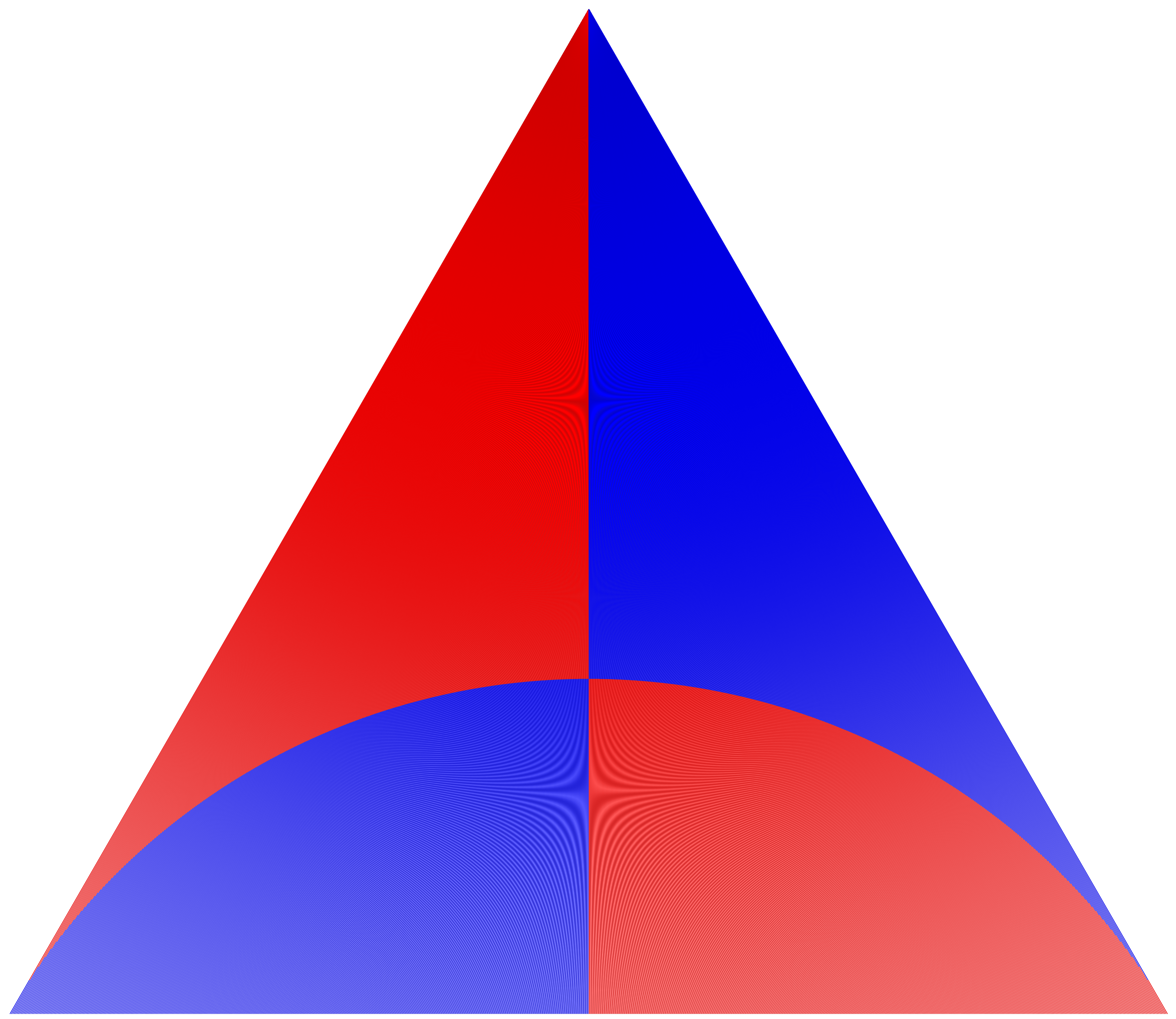
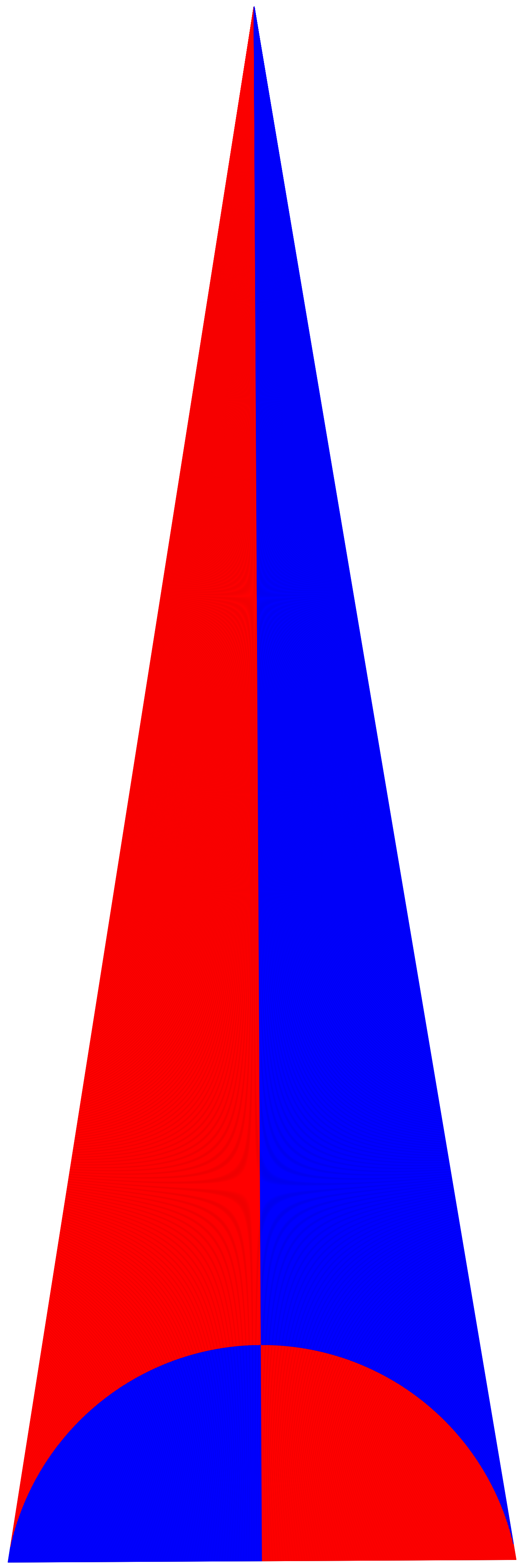
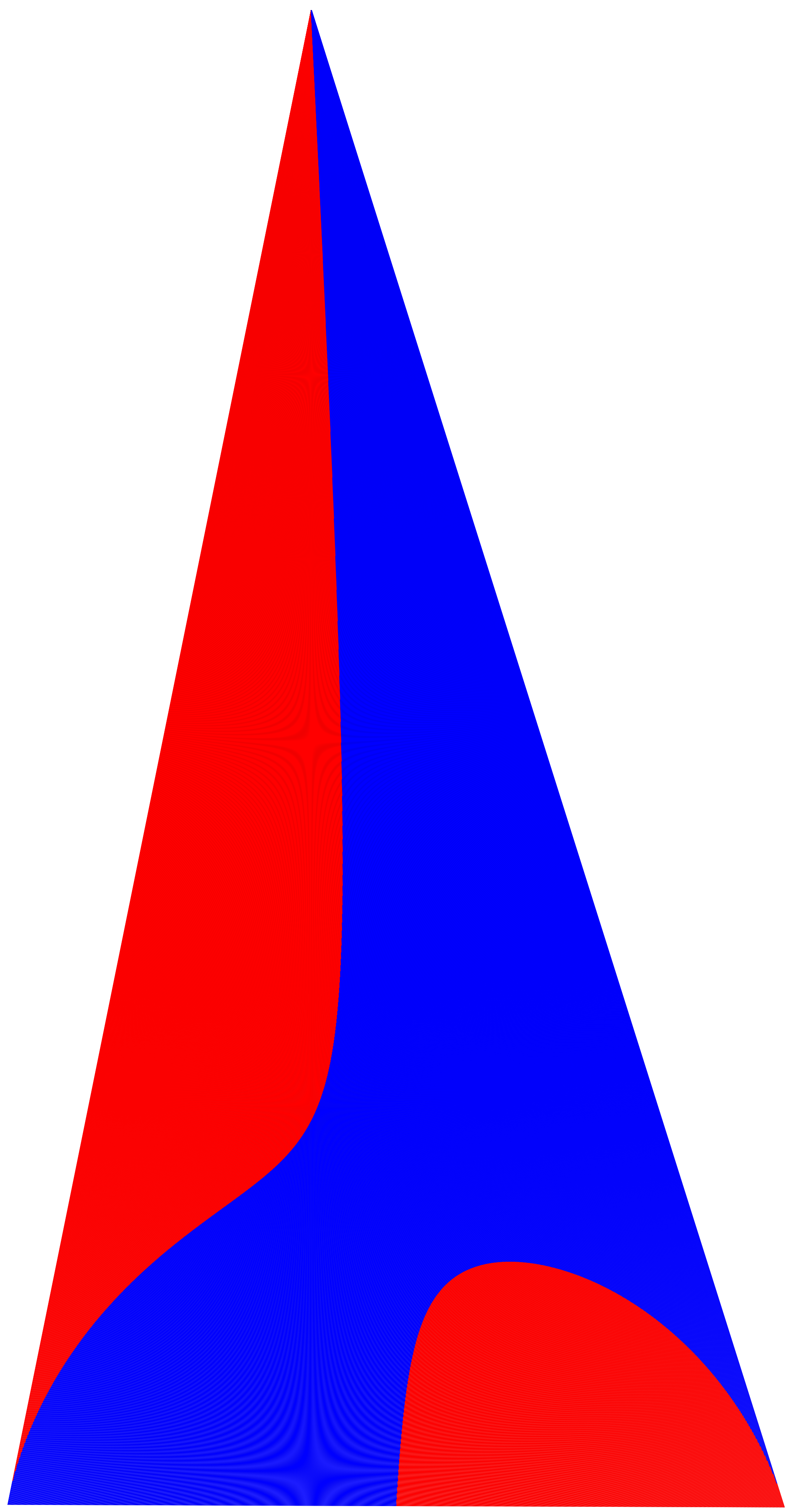


Future Work







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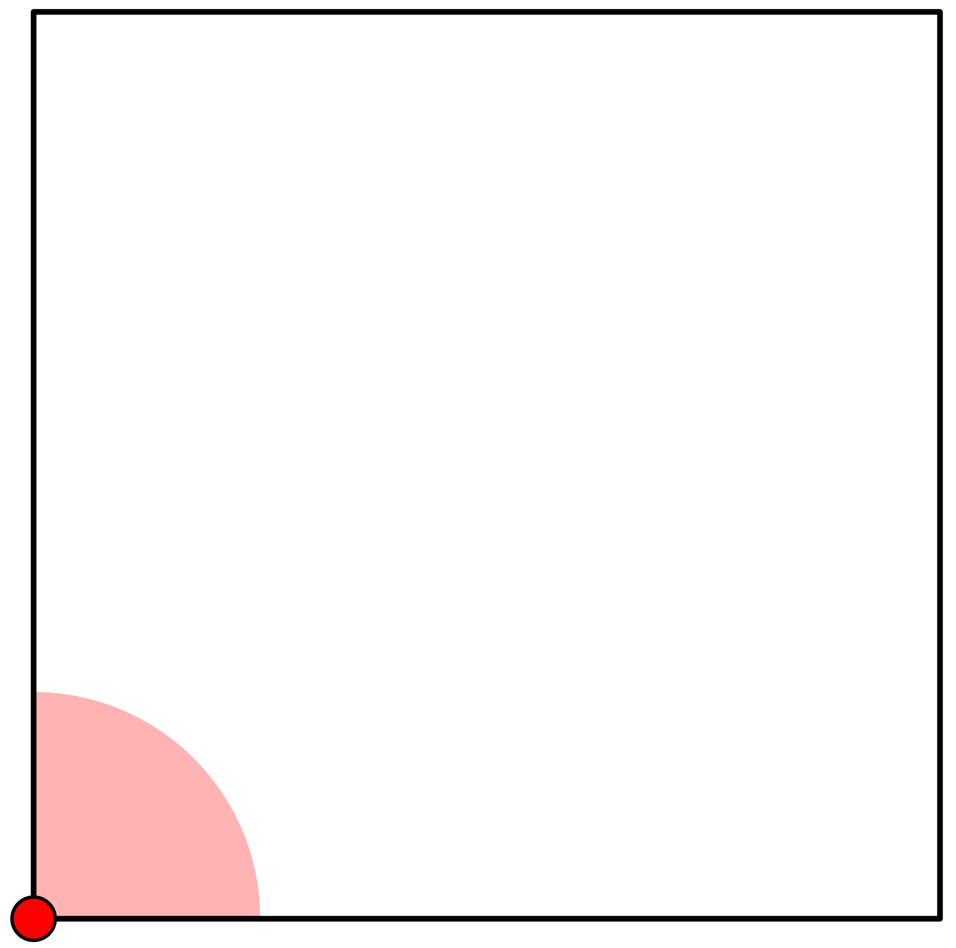
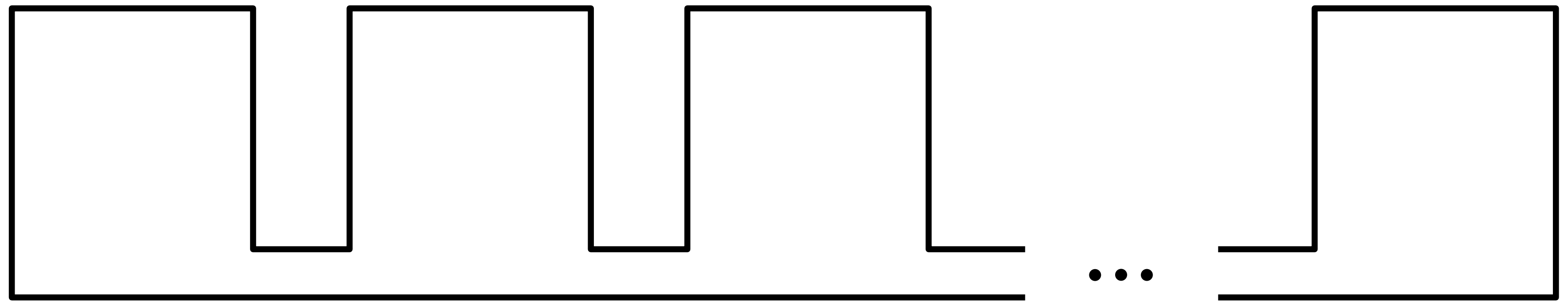
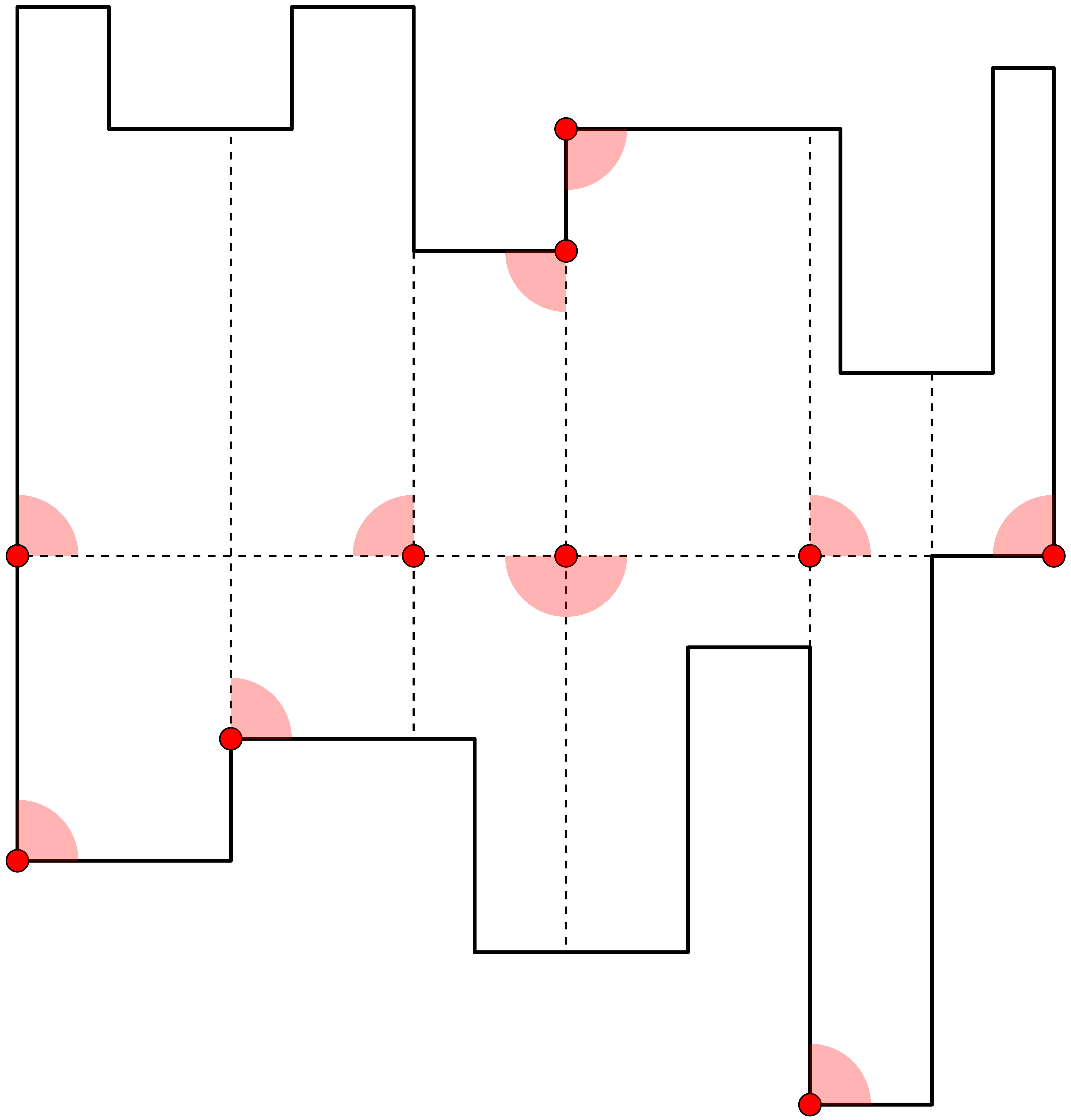




Assumed tight upper bound: $\left\lfloor \frac{n}{4} \right\rfloor \frac{\pi}{2}$



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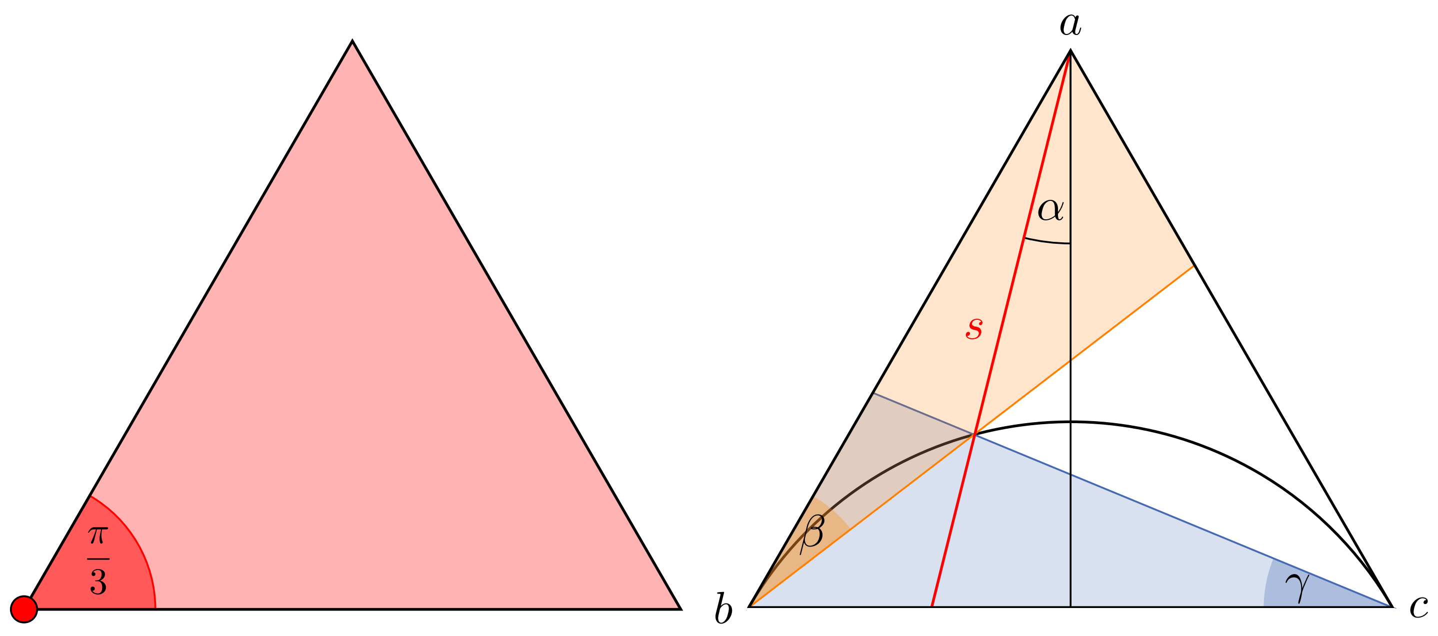
An optimal covering of an equilateral triangle has an angle of

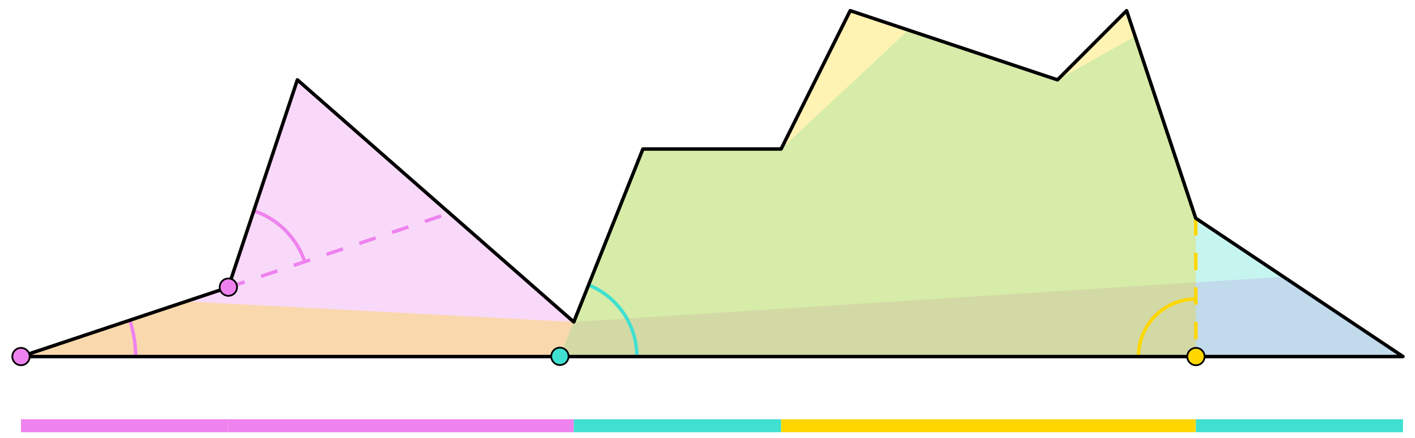
$$\frac{\pi}{3}$$



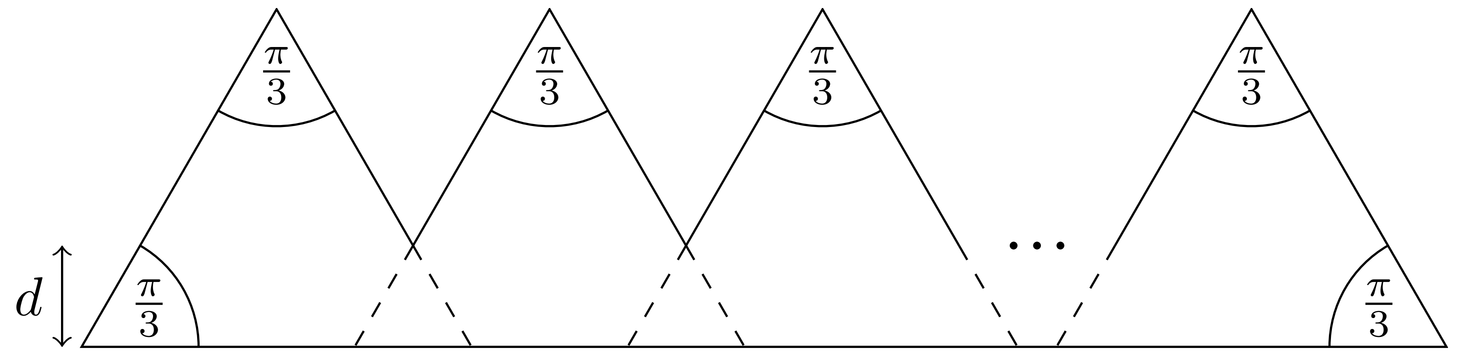
There is an upper bound of

$$(n-1)\frac{\pi}{6}$$

sufficient to cover any given histogram polygon.



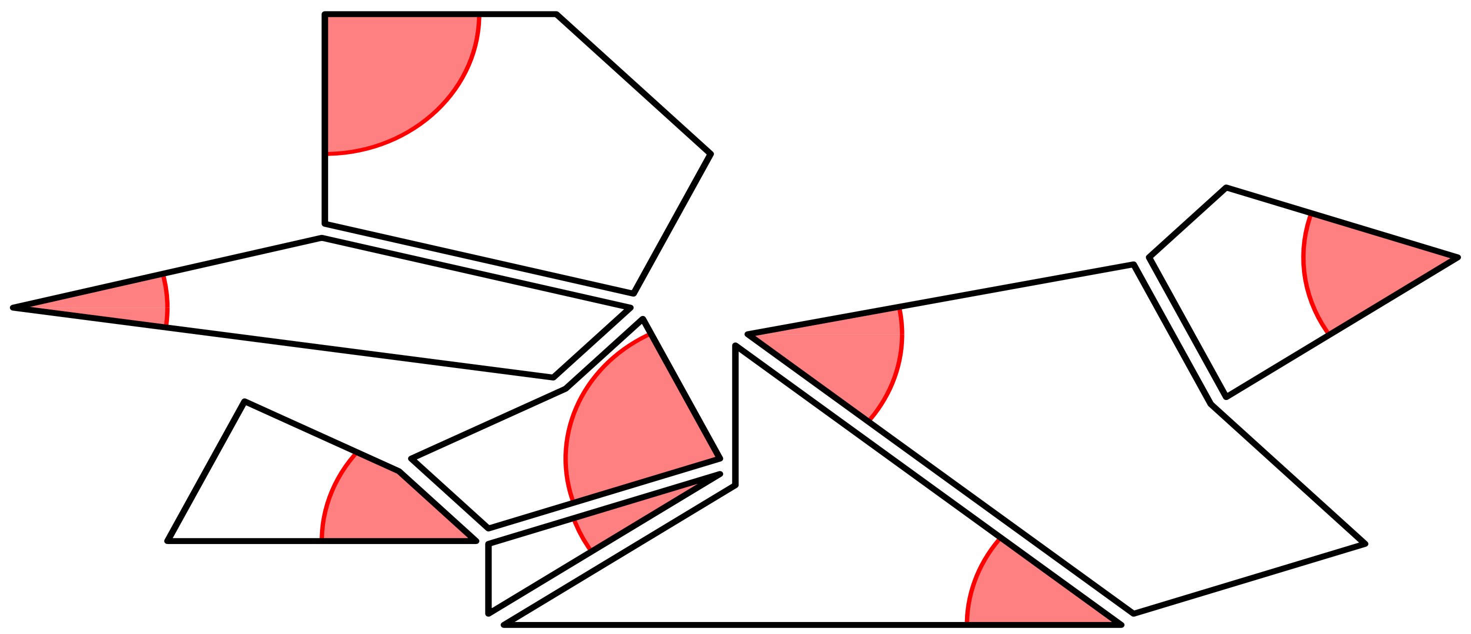
This bound is worst case optimal.



For simple polygons, we presented an upper bound of

$$(n-2)\frac{\pi}{4}$$

which may be reduced by proving a tight bound for polygons with a limited size.



A dual problem deals with independent circle packing.

We determined a duality gap of

$$\approx 4.662$$

between ICP and AAGP.